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On two arguments for fanaticism

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Abstract

Should we make significant sacrifices to ever-so-slightly lower the chance of extremely bad outcomes, or to ever-so-slightly raise the chance of extremely good outcomes? *Fanaticism* says yes: for every bad outcome, there is a tiny chance of extreme disaster that is even worse, and for every good outcome, there is a tiny chance of an enormous good that is even better. I evaluate the prospects for Fanaticism, in connection with two other kinds of general ethical principles. First, *separability* principles, which say that which option is best does not depend in strange ways on what might be going on in distant space and time—jumping off from a recent argument for Fanaticism from Beckstead and Thomas (2023). Second, *reflection* principles, about how gaining new information makes a difference to which options are best—jumping off from a recent argument for Fanaticism from Wilkinson (2022). It turns out that the situation is unstable: plausible general separability and reflection principles actually tell *against* Fanaticism, but restrictions of those same principles (with strengthened auxiliary assumptions) *support* Fanaticism. All of the consistent views that emerge are very strange.

Not madness but the mathematics
of eternity drove them.

Mary Doria Russell, *The Sparrow*

1 | ALMOST CERTAINLY POINTLESS, BUT GOOD?

> Kayla has a minor cough. She knows it is probably nothing serious—very likely it's just her seasonal allergies—but there is a small chance that it is COVID-19. She decides that she had better skip her mother's birthday celebration this year. It will already be a much smaller gathering this year than usual, but her grandfather will be there, and she doesn't want to risk giving him COVID. Still, the choice breaks Kayla's heart; she knows her mother will be deeply disappointed. Part of what makes it so frustrating is that she knows that her cough is probably nothing—so she feels like she is giving up something important for nothing.

Choices like Kayla's are frustrating; even so, sometimes it is best to give up something morally important, even though *very probably* no good will come of the sacrifice. This can happen when the stakes are high enough. Disappointing her mother is a bad thing, but not nearly as bad as giving her grandfather a deadly disease.

How far can such trade-offs take us? What if the probability of losing something important seems negligibly tiny, but what would be lost is unbearably immense—trillions of lives, whole worlds of good? Can it be worthwhile to make weighty sacrifices to avoid such risks, even though the sacrifices are almost certainly pointless?

Bostrom (2003) argues that a long-term future in which humanity successfully settles the stars would be such an immense good. This good would be lost if an existential catastrophe were to "either annihilate Earth-originating intelligent life or permanently and drastically curtail its potential" (2003, 310). Bostrom's "most conservative" estimate suggests that reducing existential risk by just one in a million million million millon would be better than saving a hundred thousand human lives directly. He concludes:

> For standard utilitarians, priority number one, two, three and four should consequently be to reduce existential risk.

Many others have been taken with this line of thought; they have also worked to generalize similar lessons beyond the concerns of just "standard utilitarians" to a broader range of ethical starting points (Ord, 2020; Greaves & MacAskill, 2021; Mogensen, forthcoming) But this conclusion is troubling. Making such a sacrifice would have a 99.99999999999999999999% chance of doing no good at all—in which case it amounts to sacrificing a hundred thousand lives for naught. This seems *fanatical*.[1]

Are such sacrifices good? Or is there some limit to how tiny the probability of doing any good can be for a given sacrifice to be warranted? The general question is whether the following principle is true.[2]

(Positive) Fanaticism. For any finite good x and for any non-zero probability p, there is some finite good y such that it is better to have y with probability p than to have x for sure.

[1] This is Bostrom's term from another context (2011), which seems to have caught on.

[2] See Section A.1 for technical background assumptions.

Bostrom's case involves sacrificing something good for a small chance of great gain. In contrast, Kayla's case involves sacrificing something good to *avoid* a small chance of great loss. This corresponds to a dual form of Fanaticism:

Negative Fanaticism. For any finite loss x and any non-zero probability p, there is some finite loss y such that it is better to have x for sure than to have y with probability p.

It is natural, though not logically inevitable, that the two theses should go together. If the goods that warrant positive fanaticism involve large numbers of happy people, or long ages of flourishing, then there are corresponding evils involving large numbers of suffering people, or eons of despair. In what follows we will primarily focus on the positive thesis, for economy of presentation.

There are two ideas built into Fanaticism: an *axiological* idea about how good things are, and a *decision-theoretic* idea about how to weigh risks. The axiological idea is that some things are extremely valuable—for Bostrom's argument, these are extremely large happy populations. The decision-theoretic idea is that an absurdly small chance of something extremely valuable can be worth a large cost. Accordingly, intuitively there are two different ways in which one might *reject* Fanaticism. One might reject the axiological thought, holding that value is *bounded*. Nothing is *good enough* to warrant some sacrifices at small odds. For example, perhaps we should exponentially discount the value of future lives, as economists often do. Alternatively, one might reject the decision-theoretic thought, holding that when it comes to "fanatical" trade-offs, the option that maximizes expected value is not best. One might say that cardinal *value* comes apart from the cardinal *utility* function whose expectation ought to be maximized—and furthermore, this utility function is bounded.[3] Or one might hold that one should round sufficiently small *probabilities* down to zero—or more generally, *discount* small probabilities to make them even smaller, perhaps infinitesimal (see Smith, 2014; Monton, 2019; Beckstead and Thomas 2020, sec. 2.3). Even if some things are extremely good, perhaps a very small chance of such a thing is not good in proportion to its chance.

It is not always easy to separate the axiological idea from the decision-theoretic idea: that requires us to make sense of a cardinal scale of value *apart* from how values are weighed in trading off risks. Some theories allow us to make sense of such a scale, but not all do. But we do not have to disentangle the two ideas in order to evaluate Fanaticism.

Why would anyone be tempted to Fanaticism? One way in is from what Bostrom called a "standard utilitarian" starting point. This package includes a *totalist* axiology, according to which extremely large happy populations are extremely good. It also includes an *expectational* decision theory, according to which the value of a chance p at getting an outcome x is given by multiplying the value of x by p. A huge number multiplied by a tiny number can still be very big.

But Fanaticism itself is not tied to either part of this specific picture. Not all ways of rejecting totalism or expectationalism are ways of escaping Fanaticism. And there are much more general arguments for Fanaticism that do not rely on Bostrom's starting point. In this essay I will focus on two closely related, interesting, and powerful arguments for Fanaticism: the argument from *strange dependence on distant space and time* from Beckstead and Thomas (2020), and the *Indology* argument from Wilkinson (2022).

[3] Or one might take on a permissive theory of risk of the kind advocated by Tarsney (2020), which says that no particular cardinal utility function is mandatory. More on this in Section 3.4.

The Fanaticism thesis concerns very large finite values. We can contrast fanatical wagers with properly *Pascalian* wagers, which intuitively have *infinite* values at stake. But it turns out that there are many tight connections between large finite values and infinite values. This essay is about those connections—and specifically how infinite lotteries make trouble for certain arguments for Fanaticism. Cases involving infinitely many possibilities or infinite values raise many paradoxes, but I am convinced that this is not a reason to ignore such cases, but rather to take them seriously and pay attention to what they can teach us. What can the basic principles of value be like, if they are not to fall into contradiction? (Furthermore, one of the two arguments I am addressing—Wilkinson's Indology argument—already relies on infinite cases.)

I do not ultimately find either of the two arguments for Fanaticism persuasive. But let's be clear: my aim here is not to settle the question of whether Fanaticism is true. I don't know. Whatever the truth of the matter, the ethics of huge numbers is deeply weird and full of surprises. This is something we must face up to. Some paradoxes of infinity are mere intellectual curiosities, but these puzzles are of real practical importance. As Hutchinson (2021) puts it :

> The future of sentient beings is potentially unimaginably large. That means if we have only a very small chance of affecting it in a lasting and positive way, taking that chance is worth it.

We have actions available that amount to taking such chances, but which involve substantial sacrifices of other important goods. For example, we might choose to divert resources that could prevent thousands of cases of malaria to instead *very slightly* reduce the risk of extreme catastrophes from climate change, or artificial intelligence, or pandemics. Since we face genuine options like this, our actual moral predicament is puzzling and troubling.

2 | STRANGE DEPENDENCE ON DISTANT SPACE AND TIME

2.1 | The argument

Beckstead and Thomas (2020, sec. 3.2) argue that if Fanaticism is false, then it turns out that which prospects are best depends on what is going on in far away places and times in weird ways.

Their argument is framed in terms of bringing into existence large numbers of happy people—though as we will see in Section 2.3, it generalizes considerably. For now, let us make the simplifying assumption that all that matters in each outcome is the total number of happy lives: any two outcomes that agree on this number are equally good. (We suppose there is no inequality: all of the different lives in question involve the same amount of happiness.)

It will be helpful to introduce some notation. If p is a probability and n is a number, write $p * n$ for a prospect that results in n happy lives with probability p, and otherwise zero lives with probability $1 - p$. For prospects X and Y, we'll use the notation $X \succ Y$ to mean that X is strictly better than Y. In this setting, we can rewrite Fanaticism like this:

> For any number of happy lives n, and for any probability $p > 0$, there is some number of happy lives N such that $p * N \succ 1 * n$.

(Beckstead and Thomas call this conclusion "Recklessness".)

TABLE 1 The "local" prospects X and Y are combined with a "distant galaxy" background prospect B.

Prospect	p	q	$1-p-q$
X	N	0	0
Y	0	n	0
B	n	0	0
$X+B$	$n+N$	0	0
$Y+B$	n	n	0

The argument is based on three ideas. The first idea is simple: it is better to have a *much higher* chance of *many more* happy lives, than a *smaller* chance of *fewer*. In symbols,[4]

More is Better. For probabilities $p \gg q$ and numbers $N \gg n$,

$$p * N \succ q * n$$

This seems hard to argue with. It follows from the idea that a much larger happy population is at least as good as a smaller happy population, which is in turn strictly better than nothing, together with very modest principles about risk.

The second idea is that the first idea is still true even if you don't know how many happy people there are in distant galaxies. For prospects X and B, let $X+B$ be a prospect that, in any state of nature s, results in the happy people that result from X in s *as well* as the happy people that result from B in s—intuitively, with the B people all living in some distant galaxy that we have no way of affecting, "in the background".

The trick is then to consider "nearby" prospects of the form $p * N$ and $q * n$, where q is much smaller than p, and N is much larger than n; we will combine these with a "distant galaxy" prospect whose uncertainty lines up with the local uncertainty in the right way: see table 1. In the first row we have a prospect X of the form $p * N$, and in the second row we have a prospect Y of the form $q * n$. In the third row we have the distant galaxy prospect B, which results in the same smaller number of happy lives n only in the case where the $p * N$ prospect succeeds. The final two rows show the result of adding together each local prospect with the fixed distant galaxy prospect.

Comparing the two combined prospects $X+B$ and $Y+B$, we observe that $X+B$ has a *slightly lower* probability p of a *much larger* number of people $n+N$. So the two ideas so far imply:

Anti-Timidity. For any probabilities $p \gg q$ and numbers $N \gg n$,

$$p * (n+N) \succ (p+q) * n$$

In other words, a chance at a sufficiently large number of happy lives is better than a *slightly higher* chance of a *much smaller* number of happy lives.

[4] The "much greater than" notation $\gg$ implicitly builds in some non-obvious quantificational structure, which is a bit complicated to spell out. But let's not quibble: I am happy to grant a simpler, stronger premise:

> For any probabilities $p > q$ and any number n, there is some number N such that $p * N \succ q * n$.

This is a consequence of Stochastic Dominance, discussed below.

The *third* idea of the argument is that Anti-Timidity implies Fanaticism. This is the central observation of Beckstead and Thomas's rich paper, which is based on a "continuum" argument (see also Wilkinson, 2022, sec. 4). Here is the basic idea. Suppose you are about to make a world with n happy lives, for sure. Then you are offered a trade: instead of just n lives for sure, you can create a *much larger* number of lives *almost* for sure. Anti-Timidity says that this is good trade: better to have chance $p = 1 - \varepsilon$ of many more happy lives than the slightly higher chance 1 of just n lives. So you take it. Then you are offered another trade: what about the slightly smaller chance $1 - 2\varepsilon$ of even *more* lives? Anti-Timidity recommends this trade, too. And so on, until you are left with a ridiculously *tiny* probability of a truly *enormous* number of lives. By transitivity, this absurdly long odds gamble is better than the sure thing you began with.

(Some deny that betterness is transitive (for example, Temkin, 2012), which would block this argument. In order to keep things under control, I will not take up this idea here. Throughout this essay I will assume without further comment that goodness is *ordered*: in particular, *at least as good* is transitive and reflexive, and *better*, *worse*, and *equally good* are related to *at least as good* in the usual ways. I do not generally assume betterness is a *complete* order—different goods may be incomparable, as we will discuss later.)

You might wonder: why so much bother in order to argue for Anti-Timidity, by way of considerations about distant galaxies? The principle already sounds very plausible without all that. The trouble is that (if betterness is an order) consistency demands either Timidity or Fanaticism—and *either one* of these is quite implausible. We face counterintuitive consequences no matter what we say. To make progress, Beckstead and Thomas explore various costs on each side. The argument we are here considering shows that strange dependence on distant space and time is one of the costs of Timidity.

The step from Anti-Timidity to Fanaticism is just math. More is Better—the idea that a much larger probability of many more happy lives is better than a much smaller probability of fewer happy lives—seems pretty unimpeachable. So the key step to examine is the move from More is Better to Anti-Timidity. What motivated this was the idea that if More is Better, then more is *still* better given arbitrary uncertainty about what is going on far away. Here is a general principle that would underwrite such reasoning:

Separability. For any prospects X, Y, and B,

$$X \succ Y \quad \text{iff} \quad X + B \succ Y + B$$

More is Better told us that $p * N \succ q * n$; then Separability tells us that $p * N$ added together with the additional gamble $p * n$ far away is likewise better than $q * n$ added together with same additional gamble—which amounts to Anti-Timidity. We can sum up Beckstead and Thomas's core idea as follows.

Theorem 1 (Beckstead and Thomas). *If all that matters is the number of happy lives, More is Better and Separability together imply Fanaticism.*[5]

Separability is a highly plausible principle. How could distant lives completely unconnected to our actions make any difference to what it is best to do about the here and now? More is Better

[5] In Section A.2 I provide an alternative more direct proof of this theorem that does not rely on Beckstead and Thomas's continuum argument.

TABLE 2 Three population lotteries. The lottery W is better than either X or Y.

Probabilities:	**1/2**	**1/4**	**1/8**	⋯
W	2	4	8	⋯
X	1	3	7	⋯
Y	1	3	7	⋯

is also hard to argue with, and the assumption that the numbers are all that matter looks like a harmless idealization. So this seems like a very strong argument for Fanaticism.

2.2 | A problem with Separability

Nonetheless, I am convinced that the argument for Fanaticism based on theorem 1 is unsound: either Separability is false, or else the numbers are not all that matters. This is due to a striking result originally proved by Seidenfeld et al. (2009), and applied to population ethics by Goodsell (2021). The argument I'll give below is an application of their proofs with minor modifications. It is closely related to the St. Petersburg paradox. As with Beckstead and Thomas's argument, for now we'll hold on to the simplifying assumption that all that matters is the number of happy lives, and so good outcomes can just be thought of as natural numbers. Later we will generalize. The upshot of the following argument is that, in this context, Separability is inconsistent with a core principle of decision theory, which I will explain below—a principle of *stochastic dominance*.

Consider a lottery for happy lives W (table 2). A fair coin is flipped until it comes up heads. If it's heads on the first flip, there are two happy lives. If it's heads on the second flip, there are four; if the third, eight; and so on.

Now consider another lottery X which has the same probabilities as W, but slightly worse outcomes: where W has probability 2^{-n} of 2^n happy lives, X has probability 2^{-n} of $2^n - 1$ happy lives. It seems clear that W is better than X.

Consider also a third lottery Y which is isomorphic to X. Then W is better than Y as well.

Now Separability tells us that two *copies* of W—one around here and the other in a distant galaxy—is better than X around here and Y in a distant galaxy. Since $W \succ X$, it is better nearby, and since $W \succ Y$, it is better far off; so $W + W$ is better than $X + Y$ all around.[6]

Now here's the trick. I told you what probabilities these lotteries assigned to their various outcomes, but I *didn't* tell you how the outcomes were arranged across states of nature. We do it in a tricky way (see table 3). As we said, the outcome of lottery W is based on flipping a fair coin until it first lands heads. In addition to those coin flips, we also flip one extra coin—the "bonus coin". For W, we simply ignore the bonus coin and get the same result however it comes up.

For X, if the bonus coin comes up heads you just get one happy life for sure, ignoring all the other coin flips. If the bonus coin comes up tails you get *twice* the outcome of W, minus one happy life. Note that this agrees with the probabilities in table 2: probability 1/2 of 1 life, 1/4 of 3, 1/8 of 7, and so on.

For Y, if the bonus coin comes up *tails* you just get one happy life, and if it comes up *heads*, then you get twice the outcome of W minus one happy life. Again, this agrees with the probabilities in table 2.

[6] To be explicit, we can do this in two steps. First, since $W \succ X$, by Separability $W + W \succ X + W$. Second, since $W \succ Y$, by a second application of Separability $X + W \succ X + Y$.

TABLE 3 How the outcomes are arranged across different states. H, n is the event where the bonus coin comes up heads, and the first heads in the St. Petersburg sequence is on the nth flip. Likewise T, n means the bonus coin comes up tails.

States:	$H,1$	$H,2$	$H,3$	⋯	$T,1$	$T,2$	$T,3$	⋯
W	2	4	8	⋯	2	4	8	⋯
X	1	1	1	⋯	3	7	15	⋯
Y	3	7	15	⋯	1	1	1	⋯
$X+Y$	4	8	16	⋯	4	8	16	⋯

TABLE 4 The lottery W strictly dominates a lottery X' which is stochastically equivalent to X.

	$H,1$	$H,2$	$H,3$	⋯	$T,1$	$T,2$	$T,3$	⋯
W	2	4	8	⋯	2	4	8	⋯
X'	1	3	7	⋯	1	3	7	⋯

Now, when these lotteries are lined up this way, what happens if you get *both* X and Y? It's the same as $W + W$! In every state, $X + Y$ and $W + W$ result in precisely the same number of happy lives. In short, since $W \succ X$ and $W \succ Y$, Separability tells us:

$$W + W \succ X + W \succ X + Y \sim W + W$$

But this cannot be.

This argument relied on some reasoning that I did not make explicit: it initially seemed clear that $W \succ X$ and $W \succ Y$—but why? Here is one way to argue for this. Consider another lottery X', which results in $2^n - 1$ happy lives if the first heads in the St. Petersburg sequence of coin flips comes on the nth flip (see table 4). Then X and X' are just rearrangements: they each have exactly the same probability of resulting in any particular outcome. That is to say, X and X' are *stochastically equivalent* prospects. So it seems clear that X and X' are equally good.

But also, W is clearly better than X': for in fact it is *sure* to turn out better than X' no matter what happens. That is, W (strictly) *statewise dominates* X. However the coin flips turn out, X' gives you $2^n - 1$ happy lives, while W gives you one more than that. So W seems clearly better than X'.

So the claim that W is better than X (and likewise Y) follows from the following two principles.

Stochastic Equivalence. Stochastically equivalent prospects are equally good.
Statewise Dominance. If Y statewise dominates X, then Y is better than X.

Could either of these ideas go wrong?

What if stochastically equivalent prospects are not equally good? This is the lesson Seidenfeld et al. (2009) drew.[7] This would mean that something else must matter for how good prospects are besides the probabilities they assign to each outcome. In fact, I do think we can imagine cases

[7] It is further developed in Lauwers and Vallentyne (2016); Lauwers and Vallentyne (2017). Bales et al. (2014) give a different argument against Stochastic Equivalence, which I discuss in Section 3 below.

where this is plausible.[8] But in the case at hand, it is hard to think of what that *something else* could be. We would need to say that when it comes to gambles that only depend on the outcome of coin flips, it can make a moral difference *which* coin flips they are. The prospect X' results in $2^n - 1$ happy lives if the first heads in the sequence of St. Petersburg coin flips is on the nth flip. But we can also consider the *extended sequence*, which starts with the bonus coin flip, and is followed by the usual St. Petersburg sequence. The prospect X results in $2^n - 1$ happy lives if the first heads in the *extended sequence* is on the nth flip. So if X and X' are not equally good, then it matters morally which of these two sequences of coin flips we use. This seems untenable. I am open to the idea that some kind of *isomorphism* between prospects which is stronger than stochastic equivalence is required to ensure that prospects are equally good. But my guess is that when we spell this notion out in any plausible way, the prospects X and X' will still count as isomorphic even in the stronger sense.

What if Statewise Dominance fails? In that case, I'm not sure what we're doing when we compare how good prospects are. As many others have emphasized (for example, Schoenfield, 2014, 268), what we ultimately care about is how well things turn out; choosing better prospects is supposed to guide us toward achieving better outcomes. In light of this, if dominance reasoning is wrong, then I don't want to be right. If A is sure to turn out better than B, then this tells us precisely the thing that betterness-of-prospects is supposed to be a guide to. A guide that does not lead us to our destination, when we already know exactly how to get there, is not worth following.

Say a prospect X *stochastically dominates* a prospect Y iff X is stochastically equivalent to a prospect that statewise dominates Y.[9] Then Stochastic Equivalence and Statewise Dominance can be combined into a single principle of *Stochastic Dominance*: if X stochastically dominates Y, then X is better than Y. Stochastic Dominance is a fairly uncontroversial principle of decision theory—even among those who reject other parts of standard expectational decision theory (such as Quiggin, 1993; Broome, 2004), and even in settings where other parts of standard expectational decision theory give out (see for example Easwaran, 2014).[10] We should not utterly foreclose giving up Stochastic Dominance—we are facing paradoxes, so some plausible principles will have to go—but I do not think this is a very promising direction. In what follows, I will take Stochastic Dominance for granted.

Theorem 2. *If all that matters is the number of happy lives, Stochastic Dominance and Separability are jointly inconsistent.*

This looks like very bad news for Separability.

Where does this leave Beckstead and Thomas's argument for Fanaticism? In theorem 1, I stated a version of their argument which has Separability as a premise. If Separability is false, that version is unsound. But Beckstead and Thomas are more cautious. They write:

[8] For example, the probability of an ideally sharp dart hitting a particular point may be zero—but the prospect of sparing a child from malaria if the dart hits that point (and otherwise nothing) may still be better than the prospect of getting nothing no matter what. But these two prospects are stochastically equivalent. Perhaps what is best depends on what features of its outcomes are *sure*—where in general this can come apart from what is *almost sure*—that is, has probability one.

[9] There are subtleties about the definition of stochastic dominance: see Russell (forthcoming).

[10] For other defenses of Stochastic Dominance, on which I here have drawn, see Tarsney (2020, 8); Wilkinson (2022, 10); Bader (2018).

> The argument … is closely related to well-known arguments from 'separability' for totalist views in population ethics (see Broome, 2004). However, the issue for us is not separability in general—perhaps modest violations of separability would be acceptable—but the particular dramatic violations to which timidity leads. (2023, footnote 15 on p. 17)

I'm not sure what they have in mind when they speak of "modest" versus "dramatic" violations of separability. But it is entirely true that it might turn out that, while Separability has counterexamples, the cases that Beckstead and Thomas's argument relies on are not among them. Still, I think if we conclude that Separability simply *can't* be true in general, we should lose much of our confidence in the particular judgments as well. That would tell us that what is better than what really *does* depend in strange ways on what is going on in distant space and time. Given a choice between the lotteries W and X, it *matters* whether you think there is another St. Petersburg population lottery going on in a distant galaxy. This is bizarre—but Stochastic Dominance tells us that it is true. So our intuitions about Separability, while admittedly strong, are not to be trusted.

Is there a more restricted principle than Separability which has better hope of being true, and which still can underwrite Beckstead and Thomas's argument? Here is something to try. A *simple* prospect is one that has only finitely many possible outcomes.

Simple Separability. For any simple prospects X, Y, and B, $X \succ Y$ iff $X + B \succ Y + B$.

This very restricted principle is consistent with Stochastic Dominance. And Simple Separability can do the same work as Separability in Beckstead and Thomas's argument.

But is it true? It might be, but it is hard to be confident of this. What would the motivation be for it that is not also motivation for the unrestricted principle? It can't be simply the idea that if what is going on in distant space and time is the same for both of two options, then it is irrelevant to which is better. That idea supports full-fledged Separability. So is there something special about *simple* prospects that makes their value insensitive to what is going on in distant space and time? I leave this question open.

2.3 | Beyond the numbers

There is one other possible response to the preceding arguments: perhaps the idealizing assumption that the number of happy lives is all that matters is not an innocent simplification. Both theorem 1 and theorem 2 rely on the drastic idealization that lives can be freely rearranged between near and far galaxies without affecting anything of value—for instance, there are no morally important relationships between people in the same galaxy. Could this be where we went wrong? This would not save Beckstead and Thomas's argument, but it might allow us to salvage Separability.

In fact, both arguments can be generalized to avoid relying on this drastic idealization. I have talked about adding up *numbers* of happy lives—but all the arguments really require is a much more general sense in which outcomes can be "added up". The key idea is that each finite outcome can be split up into two parts: a *near* part, concerning what is going on around here in the part of the world we might make any difference to, and a *far* part, concerning what is going on in Beckstead and Thomas's "distant galaxy". We can "add up" a *near outcome* x and a *far outcome* y

to get a combined outcome which we'll call $x \oplus y$. We can similarly talk about *near prospects* and *far prospects*, which can be "added up" outcome by outcome.

Separability can be restated in these more general terms.[11]

Separability. For any near prospects X and Y, and any far prospect B,

$$X \succ Y \quad \text{iff} \quad X \oplus B \succ Y \oplus B$$

For any far prospects X and Y, and any near prospect B,

$$X \succ Y \quad \text{iff} \quad B \oplus X \succ B \oplus Y$$

The strong idealizing assumption that all that matters is the number of happy lives can then be replaced with much weaker structural assumptions about adding up outcomes. If all that matters is the number of happy lives, then we can freely rearrange happy lives between the near and far parts of the world without losing any value: an outcome with m happy lives nearby and n happy lives far away is just as good as an outcome with *zero* happy lives nearby and $m + n$ happy lives far away. In our more general setting, the key principle is that we can always *compensate* somehow for making things worse nearby, by making things sufficiently better far away (and vice versa). We will call this assumption *(Positive) Compensation*; it is stated precisely in Section A.1. (We will encounter a *Negative* Compensation principle in Section 3. When I use "Compensation" without qualification, I mean Positive Compensation.)

We could also restate the other premise of Beckstead and Thomas's argument, More is Better, in these more general terms. But if we help ourselves to the stronger assumption of Stochastic Dominance, we can avoid some complications in the general statement of the theorem and also give a substantially simpler proof (which is in Section A.2).

Theorem 3. *Stochastic Dominance, Simple Separability, and Compensation together imply Fanaticism.*

This generalization of theorem 1 does not provide a new way of defending Beckstead and Thomas's argument for Fanaticism: for we can also generalize theorem 2.

Theorem 4. *Stochastic Dominance, Separability, and Compensation are jointly inconsistent.*

(Proofs are given in Section A.2, A.3.)

So to argue for Fanaticism on the basis of theorem 3, we would again have to find a way to motivate *Simple* Separability without going all the way to full-fledged Separability.

But the generalization also clarifies how full-fledged Separability might still be true after all: it might be *Compensation* that fails, instead. There is more that matters morally than just the total number of happy lives; lives can not be freely rearranged without any effect on value.

Note that, unlike Stochastic Dominance or Separability, Compensation is not a principle about risk, but purely about what ways for the world to be are best. Giving up Compensation imposes constraints on axiology. Rejecting Compensation will be strange—but theorem 4 ensures that *every* consistent view is strange. One way of developing this idea is to say that eventually the value

[11] We have added a second clause because our new notion of "adding up" outcomes or prospects need not be commutative.

of a galaxy is "saturated", so no further vast improvements are possible—and in particular, no further improvements would suffice to make up for a large loss of value in another galaxy. Adding more happy lives to a galaxy far away that already contains some huge number of happy lives simply cannot make up for eliminating many happy lives nearby.

Here is a simple model. "Near" value and "far" value are represented by two bounded utility functions. The total utility of an outcome is given by the sum of its near utility and its far utility, and the best prospect is that which maximizes expected utility. In this model, if "far utility" is already close to its upper bound, there will be no way of improving it enough to compensate for a large loss in "near utility". This model satisfies Stochastic Dominance and Separability, but not Compensation. Fanaticism also fails in this model: since total utility is bounded, there are no goods immense enough to warrant extremely long-odds gambles. (One way of implementing this idea is social discounting. Concretely, suppose each galaxy is inhabited by at most countably many individuals, ordered somehow; each individual's welfare is represented by a number in $[0, 1]$; the utility of a galaxy is given by adding up exponentially discounted individual welfare.)

It turns out that *every* natural model is going to work out in basically the same way. It is possible to show that Stochastic Dominance and Separability, together with some auxiliary assumptions, imply that Fanaticism is *false*.[12] A more precise statement and proof is given as theorem 8 in Section A.3.

Where does this leave us? (I'll hold Stochastic Dominance and the order axioms fixed.) We find ourselves in an unstable dialectical situation. The basic idea, remember, was that the value of a prospect shouldn't depend in strange ways on distant space and time: if two prospects are exactly alike in terms of the probabilities they assign to distant goings-on, then it seems we should be able to "subtract" the distant part and compare the prospects just based on what they say about the part of the world where the choice between them might conceivably make a difference. What we have found is that a *restricted* version of this idea—*Simple* Separability—together with the "value rearrangement" Compensation principle and Stochastic Dominance, implies that Fanaticism is true. But Separability *in general* is *inconsistent* with Compensation, and in fact (given some modest auxiliary assumptions) it implies that Fanaticism is *false*.

3 | INDOLOGY

3.1 | The argument

Wilkinson (2022) gives another argument for Fanaticism which is closely related to Beckstead and Thomas's. This argument is the last of three in Wilkinson's rich paper, and the one that Wilkinson reckons the most compelling (p. 450). (My presentation generalizes Wilkinson's.)[13]

This argument also has three steps. Once again, we will consider outcomes that can be split into two parts, which we'll call "near" and "far."

[12] The first auxiliary assumption says that there is a symmetry between near and far outcomes. The second is a condition on the ordering of outcomes; a sufficient condition that is much stronger than what we need is that this order is complete. I discuss issues about incomparability in Section 3.2.

[13] Wilkinson's presentation presupposes totalism, which allows him to state Background Independence in terms adding the *(cardinal) values of outcomes* together—as represented by real numbers. I am generalizing his argument to a more axiologically neutral setting.

Step 1. Consider a different way or restricting Separability, where we require that the "background" prospect involves no uncertainty.

Background Independence. For any near prospects X and Y and any far outcome b,

$$X \succ Y \text{ iff } X \oplus b \succ Y \oplus b$$

Say b is an outcome where there are a million happy lives, far away and long ago. Then $X \oplus b$ is a prospect that is sure to result in a million *additional* happy lives (far away) besides what results from X (around here). Background Independence is the basis for the classic "Egyptology" objection to the average view in population ethics (McMahan, 1981, 115; Parfit, 1984, 420). It would be very strange if, when choosing between two policies, our decision might turn on whether an additional million people thrived in ancient Egypt, utterly unaffected by our choice.

We might worry about whether this principle is true, for the kind of reasons discussed in the previous section. But at least it does not give rise to the exact same kind of problems as the general Separability principle. Since the background outcome a does not build in any uncertainty of its own, there is no worry about this interacting with X and Y in different ways in different states, which might compensate for Y's bad cases without compensating for X's bad cases.[14]

Step 2. Wilkinson gives a different argument for a kind of separability failure, based on Tarsney (2020). Suppose Fanaticism is false. Then it can be shown that there is a *risky lottery* X and a *safe lottery* Y such that $X \not\succ Y$, but for some *background prospect* B, which is independent of both X and Y, $X \oplus B$ stochastically dominates $Y \oplus B$. Then Stochastic Dominance tells us:

$$X \oplus B \succ Y \oplus B$$

Instead of imagining extra lives in ancient Egypt, Wilkinson imagines the background prospect concerning what went on in the ancient Indus valley, about which we have a great deal of uncertainty.

> We happen to know even less about what happened in the ancient Indus Valley than in ancient Egypt—archaeological research and excavations of key sites in India began centuries later than similar work in Britain, Italy, and Egypt. So there is likely plenty left to learn in Indology. (Wilkinson, 2022, 474)

Step 3. Now consider how things might go once we *remove* this uncertainty. The effect would be to replace the uncertain prospect B with some particular outcome b. But since $X \not\succ Y$, Background Independence tells us

$$X \oplus b \not\succ Y \oplus b$$

This is strange!

> From Background Independence we know that, whatever you might uncover in your research, you would conclude that the risky lottery is no better than the safe lottery.

[14] Furthermore, unlike Separability, Background Independence is consistent with Stochastic Dominance, even if only the numbers matter. Thanks to Zach Goodsell for discussion.

> … So you know what judgement you would make if you simply learned more, no matter what it is you would actually learn. So why do the many years of research?
>
> … Surely we can sidestep those years of research into how B turns out, and make the judgement required by every possible value of b. Surely rationality requireWilkinsons that we do so, rather than require that we do not. But, if we deny Fanaticism, we must accept this inconsistency — an inconsistency which, to me, seems far more absurd than simply accepting Fanaticism and even more absurd than the Egyptology Objection. (Wilkinson, 2022, 475)

I take it that Wilkinson's judgment here is based on the following principle.

Negative Reflection. For prospects X and Y and a question Q, if X is not better than Y conditional on any possible answer to Q, then X is not better than Y unconditionally.[15]

This is a kind of "no regret" principle (compare Arntzenius, 2008; see Russell and Isaacs, 2021, sec. 2, and references therein). If Negative Reflection is violated, then you may choose the better prospect, go on to do your research, and then find that your chosen option isn't better after all—no matter what your research turns up. That does seem weird.

As with Beckstead and Thomas's argument, Wilkinson's argument can be summarized by a theorem. Before I state it in general terms, there is one structural point to explain. An important feature of the background prospect B that figures in the Indology argument is that it has "heavy tails," allocating substantial probability to very good outcomes, and also to very *bad* possible outcomes. The results in Section 2 only relied on very good outcomes; *Positive* Compensation ensured that there were sufficiently good outcomes around. For the Indology argument we additionally need a *Negative* Compensation principle, which ensures that are sufficiently *bad* outcomes. It is spelled out in Section A.4.[16]

Here is the generalization of Wilkinson's main result.

Theorem 5. *Stochastic Dominance, Negative Reflection, Background Independence, and Positive and Negative Compensation together imply Fanaticism.*

These four principles seem very plausible, and the Indology argument seems very strong. But there are two problems with it—one big, and one smaller.

[15] We model a *question* as a *regular partition*, a set of mutually exclusive and jointly exhaustive events each of which has positive probability. Following Tarsney, Wilkinson's original way of running the argument used a continuous distribution B, but using a discrete distribution is simpler and avoids technical problems that arise if we do not require that the "possible answers" all have positive probability (see Arntzenius et al., 2004).

[16] Wilkinson appealed to the stronger principle that composing near and far outcomes just amounts to adding up real numbers—which follows from his totalist assumptions. Tarsney's theorem, which Wilkinson's argument relies on, uses the technical assumption that outcomes have *additive conjoint structure* (see Tarsney, 2020, 11; Krantz et al., 1971, 245–66). Positive and Negative Compensation both follow from one of the four axioms that characterize such structures.

TABLE 5 Y dominates X, X is stochastically equivalent to X', but by Negative Reflection, Y is not better than X'.

	Heads	Tails
X	Hike	Show
Y	Hike + Sticker	Show
X'	Show	Hike

3.2 | Consequences of Reflection

Here is the big problem:

Theorem 6. *Stochastic Dominance and Negative Reflection together imply that Fanaticism is false.*

This tells us that the premises of the Indology argument—which include Stochastic Dominance and Negative Reflection—are in fact jointly inconsistent, and so the argument cannot be sound.

Here is the idea behind theorem 6. (Details are in Section A.5.) First, Fanaticism implies the existence of *generalized St. Petersburg prospects* (see Beckstead and Thomas, 2023, sec. 4). We can find a sequence of outcomes that get better very fast, and use these to construct a prospect which is *strictly better than any of its outcomes*; Russell and Isaacs (2021) call such prospects *improper*. The existence of improper prospects is a particularly unsettling consequence of Fanaticism.

Second, improper prospects violate Negative Reflection.[17] We can play *two* generalized St. Petersburg games X and Y, independently. We can choose Y so its outcomes are a little better than X's—and so Y stochastically dominates X—but still, none of Y's outcomes are as good as the *prospect* X. Conditional on any way Y could turn out, Y is only as good as one of its mundane outcomes, and so no better than X. But Y is unconditionally better than X, contradicting Negative Reflection.

There is also a second, less serious problem for the Indology argument. Many people hold that some outcomes are *incomparable* to one another: neither is strictly better than the other, but they are still not equally good (for instance, Chang, 2002). Is it better to experience a profoundly moving improvised one-person show, or to take a meditative three-day wilderness hike in the Sierra Nevada? Neither seems clearly better than the other. One way of arguing that they are also not equally good is that *sweetening* either option, say by throwing in a free sticker, still does not make one option seem better than the other. But the combination of Stochastic Dominance and Negative Reflection rules out such cases (see Hare, 2010; Schoenfield, 2014; Bales et al., 2014; Bader, 2018).

The argument is based on so-called "opaque sweetening" (see table 5). You flip a coin: Heads, you take the hike; Tails, the show. Call this prospect X. Then consider a "sweetened" prospect Y: Heads, you get the hike *and* a sticker; Tails you get the show. The sweetened option Y dominates X, so Y is better than X. But then you think, what's so special about Heads? Consider the prospect X': Heads, you go to the show; Tails, you take a hike. Then X and X' are stochastically equivalent—so equally good. In short, Y *stochastically dominates* X'—so Y is better than X'. Finally, though, by

[17] The analogous point for *Positive* Reflection (below) was one of the lessons of the two-envelope paradox (see Broome, 1995; Chalmers, 2002).

TABLE 6 A simpler Separability argument, using Stochastic Dominance.

	p	q	q	⋯	q
x	x	x	x	⋯	x
Y	y	0	0	⋯	0
B	0	z_1	z_2	⋯	z_n

TABLE 7 The generalized counterexample to Separability.

	E_1	E_2	E_3	⋯	F_1	F_2	F_3	⋯
X	0	0	0	⋯	x_1	x_2	x_3	⋯
Y	y_1	y_2	y_3	⋯	0	0	0	⋯
W	w_1	w_2	w_3	⋯	w_1	w_2	w_3	⋯
Z	z_1	z_2	z_3	⋯	z_1	z_2	z_3	⋯

assumption Y is *not better* than X' given Heads, nor is it better given Tails. So Negative Reflection implies that Y is not better than X'. We have a contradiction.[18]

So the second, smaller problem for the Indology argument is that two of its premises—Stochastic Dominance and Negative Reflection—are inconsistent with incomparability between outcomes.

This consequence is especially pressing in the context of an argument for Fanaticism: for Fanaticism provides special reasons to suspect that some *prospects* are incomparable with one another—even in a context where all possible *outcomes* are totally ordered; but a very similar argument from Negative Reflection rules this out as well.[19] Lauwers (2016) shows that if outcomes are represented by unbounded real-valued utilities, then there is no *constructible* total ordering of prospects that obeys an independence axiom (similar to the Sure Thing Principle, discussed below). We could never hope to write down a decision theory, extending the standard theory of expected value, that told us how to rank every pair of prospects—the existence of such orders in platonic heaven depends on the Axiom of Choice.[20]

Despite this, my preferred response to the "opaque sweetening" argument is to reject incomparability (see Dorr, Nebel, and Zuehl, forthcoming)—but I do not wish to take this controversial view for granted in this essay. Others have wielded this as an argument against Stochastic Equivalence (Schoenfield, 2014; Bales et al., 2014). A more standard view gives up Negative Reflection (for example, Aumann, 1962; see also Bader, 2018).

In contrast, consider the analogous principle for *at least as good* rather than *not better*:

Positive Reflection. For prospects X and Y and a question Q, if X is at least as good as Y conditional on any possible answer to Q, then X is at least as good as Y unconditionally.

[18] Note also that this argument does not turn on the infinite partitions that figure in both theorem 5 and theorem 6.

[19] The generalized argument requires some additional premise that allows us to replace the outcomes Hike and Show in the argument with uncertain prospects: the Sure Thing Principle (Section 3.3) suffices.

[20] Another consideration is that Askell (2018) shows that for *infinite* populations, no total ordering of outcomes is compatible with both the principle that what is better for everyone is better overall, and the principle that betterness is invariant under arbitrary permutations of welfare.

Unlike Negative Reflection, Positive Reflection is perfectly compatible with incomparability, whether between outcomes or prospects (in the presences of Stochastic Dominance). But Positive Reflection still *does* rule out Fanaticism (again, in the presence of Stochastic Dominance). The argument is essentially the same as for theorem 6.

To sum up, not only do Reflection principles fail to support Fanaticism, but in fact they undermine it.

Still, I agree with Wilkinson that Reflection principles have much to be said in their favor. One argument is based on the *value of information*.[21] Suppose you have two basic options—*Take It* or *Leave It*. Then consider a third option: *Take It If the Taking Is Good*. That is, you can commit to Taking It *only* if, given the additional information of how many people there are in the universe, Taking It turns out to be as good as Leaving It. This third option seems like it should be at least as good as either of the other two. It amounts to doing what is best given *more* information, rather than less—and how could this be bad? Betterness-for-prospects is supposed to be a guide to outcomes that really *are* better, given *all* the information about how things turn out. Betterness-given-more-information should be as good or better a guide than betterness-given-less-information. As Broome (1991, 129) puts it:

> [P]robabilities derived from more information have a higher status than those derived from less. … At the extreme, what would actually happen has the highest status of all. … This at least is true: you ought not to found your judgements on lower-status probabilities when higher-status probabilities are available.

But if Positive Reflection fails, then it can turn out that Taking It If the Taking Is Good is *not as good* as just Leaving It. For if Positive Reflection fails, then it could be that *Take It* is as good as *Leave It* given *any* number of people—and so *Take It If the Taking Is Good* simply amounts to the same thing as *Take It*—and yet *Take It* is *not* as good as *Leave It*. This seems absurd.

Such arguments for Positive Reflection amount to arguments *against* Fanaticism. Indeed, Positive Reflection, together with other standard axioms of decision theory, presses us to a specific kind of anti-fanatical view: expected utility maximization with a bounded utility function (see Hammond, 1998; for a generalization see Russell, 2020). This kind of theory keeps Stochastic Dominance and both Positive and Negative Reflection. Which of the other two propositions in the inconsistent set we keep—Background Independence or Compensation—depends on how we fill in details. As in Section 2.3, we could let the utility of an outcome $x \oplus y$ be given as the sum of two bounded utility functions $u_1(x) + u_2(y)$—one "near" and one "far". This version satisfies Background Independence (and indeed Separability), but not Compensation, as we discussed earlier. The alternative is to use a utility function that is not additively separable in this way: for example, instead of taking a sum of two bounded functions, we could use a bounded function of the *sum* of the two parts. That is, if we have functions f_1 and f_2 that represent near and far outcomes with numbers, respectively, then the utility would be $u(f_1(x) + f_2(y))$ for some bounded function u. This kind of model keeps Compensation, while giving up Background Independence.

[21] This is based on one of several arguments Russell and Isaacs (2021) give in defense of Positive Reflection, generalizing standard arguments for orthodox expectational decision theory in finite cases. Their arguments are about rational preference, but they can be easily adapted to arguments about which prospects are morally best. Note that while this argument is structurally related to standard *dynamic consistency* arguments, it is not an argument about sequential decision-making through time.

But even though Reflection principles have much in their favor, we should not be too hasty to accept them, along with their anti-Fanatical consequences. For these consequences are also very strange.

3.3 | The Sure Thing Principle

As in Section 2.2, we should ask whether we can repair Wilkinson's Indology argument by substituting some more restricted reflection principle that can do the same work. Here is something to try.[22]

The Sure Thing Principle. If E has positive probability and prospects X and Y are equally good conditional on not-E, then X is at least as good as Y conditional on E iff X is at least as good as Y unconditionally.

This principle can be motivated by very similar reflection considerations.[23] Say you are again choosing between *Take It* or *Leave It*. And you are about to learn whether it's *Hot* or *Cold*. Suppose that, given Hot, Take It is at least as good as Leave It; and given Cold, Take It is just as good as Leave It. Then whatever you learn, you will conclude that Take It is at least as good as Leave It. So it seems you ought to be able to conclude *in advance* that Take It is at least as good as Leave It—this is the same kind of reasoning as in Wilkinson's Indology story. Similar reasoning supports the converse direction of the Sure Thing Principle as well.

The Sure Thing Principle is somewhat controversial (more so than Stochastic Dominance). Theories that allow risk aversion violate it (Quiggin, 1993; Buchak, 2013); theories that tell us to ignore small-probability outcomes do as well.[24] But, while it is in the same spirit as Positive and Negative Reflection, it is much less demanding. Repeated applications of the Sure Thing Principle can tell us that if one option is better than another conditional on each event in a *finite* partition, then it is better unconditionally. But Positive and Negative Reflection apply even to *infinite* partitions—and on these the Sure Thing Principle is silent. Unlike the infinitary principles, the Sure Thing Principle is consistent with Fanaticism. It is also consistent with incomparable prospects.

Can the Sure Thing Principle, together with Background Independence, serve as the basis for a new "Egyptology" style argument for Fanaticism? Not in the same way as in Wilkinson's Indology argument, no. The background prospect B that figures in Tarsney's construction, which Wilkinson's argument is based on, has infinitely many possible outcomes—recall its "heavy tails". (See Section A.4 for more explanation.) But we can give a different argument. The Sure Thing Principle restricts the space of possible decision theories quite a bit. Meanwhile, Background Independence restricts the space of possible axiologies quite a bit. Together, these squeeze us toward Fanaticism.

[22] There are several non-equivalent principles that are called "The Sure Thing Principle"—see Schlee (1997). But I think this is one reasonable candidate for that label. In Savage's framework probabilities are not given; we can replace the condition that E has positive probability with the condition that E is "non-null" in the sense that some prospects that agree on $\neg E$ are not equally good.

[23] Indeed, this is how Savage (1954, sec. 2.7) motivated it.

[24] Suppose E is an event with positive but negligibly small probability. Let X and Y be prospects that always have the same outcome, except in E, in which case Y is sure to turn out strictly better than X. Then theories that neglect negligibly small probabilities will say that X and Y are equally good unconditionally—so $X \gtrsim Y$. But while X and Y are equally good conditional on not-E, X is not at least as good as Y conditional on E.

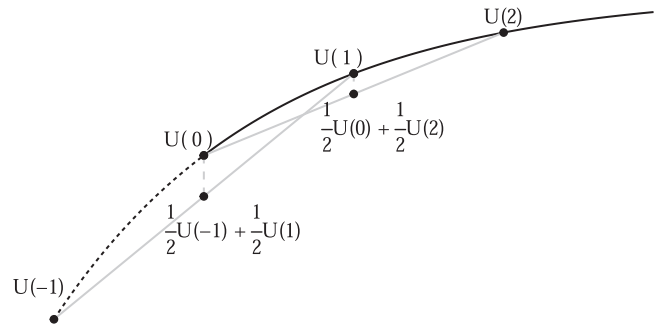

FIGURE 1 If U is concave for positive values, Background Independence requires that U is also concave for negative values, which leads to Negative Fanaticism.

I noted earlier that the Indology argument relies on *bad* outcomes, in a way that the Separability arguments in Section 2 did not. We can show that the argument would not work without them, with a model. Suppose that all that matters is the number of happy lives. Then we can construct an expected utility model with a bounded utility function (which takes values between −1 and 0):

$$U(n) = -2^{-n}$$

This is a theory according to which additional happy people have diminishing marginal utility. Any bounded expected utility model satisfies Stochastic Dominance and both Positive and Negative Reflection (see Hammond, 1998). Moreover, *this* utility function has the nice feature that, for a prospect X,

$$EU(X + n) = 2^{-n} \cdot EU(X)$$

This is an order-preserving transformation of $EU(X)$, which guarantees Background Independence. Since the utility function is bounded, this model does not satisfy Fanaticism.

But we cannot simply extend this utility function down to negative numbers representing unhappy lives. Note that $U(n)$ bends downward: for instance,

$$U(1) > \frac{1}{2}U(0) + \frac{1}{2}U(2)$$

This means that in this model, getting the one happy life for sure is better than flipping a coin to decide whether you get zero or two. In other words, the model endorses *risk aversion* with respect to happy lives. But Background Independence tells us that we can "shift" this risk-averse preference down, by adding any negative value to each of the outcomes. So we must also have, for example,

$$U(0) > \frac{1}{2}U(-1) + \frac{1}{2}U(1)$$

(See figure 1.) In fact, the utility function must continue to bend downward no matter how far you go to the left. This means that its extension to negative values must be unbounded below—which results in *Negative* Fanaticism, where an arbitrarily small risk of a *very bad* outcome is worse than a certain outcome which is still quite bad.

This idea generalizes. As it turns out, the general argument needs one more premise. As I noted earlier, we have not generally assumed that prospects are comparable: there may be prospects such

that neither is better than the other, and yet they are not equally good. For the present argument we still do not need to assume comparability in general—but we do require some especially simple value comparisons. As throughout this paper, we call prospects "good", "bad", or "neutral" insofar as they are better, worse, or equally good as the designated "zero" outcome.

Simple Comparability. If x is a good outcome and y is a bad outcome, then a fair lottery with outcomes x or y is good, bad, or neutral.

Now we can state the new "Egyptology" argument for Fanaticism.

Theorem 7. *The Sure Thing Principle, Stochastic Equivalence, Background Independence, Positive and Negative Compensation, and Simple Comparability together imply that at least one of Positive Fanaticism or Negative Fanaticism is true.*

A proof is given in Section A.6.

3.4 | Scorecard

What are we to make of this? Once again, the dialectical situation is unstable. A strong reflection principle implies that Fanaticism is false. But a weak reflection principle—the Sure Thing Principle—together with Background Independence (and some auxiliary principles) implies that Fanaticism is true. Facing both opposing arguments, I do not think we should place our trust in either yet. Whatever the truth, it is very strange, and we are still far from understanding it.

What are our options? (In order to keep this list under control, I will hold Stochastic Dominance and the order axioms fixed.)

1. Accept at least one of Positive or Negative Reflection, and thereby reject Fanaticism. Then we must also give up one of Background Independence or Compensation; things are strange either way. If Background Independence fails, then we have even *stranger* dependence on distant space and time than before—since now even if we *know* precisely what is going on out there, it *still* can make a difference to what prospects for nearby matters are best. If Compensation fails, then a part of the universe can mysteriously run out of room for more value (or disvalue), and rearranging populations between near and far parts of the world can make a moral difference.
2. Reject Positive and Negative Reflection, but keep the Sure Thing Principle. This is strange, first because it seems unprincipled, when the arguments in support of each are so similar—including arguments based on regret or the value of information. Second, because again we must either reject Background Independence or Compensation, or else accept some form of Fanaticism—each of which is strange.
3. Reject Reflection *and* the Sure Thing Principle, perhaps going for some less standard decision theory (such as risk-weighting or discounting small probabilities). This allows for regret and negative value of information even in simple cases.[25] Fanaticism remains an open question.
4. Give up Simple Comparability. This might sound desperate—but Tarsney (2020) defends the austere view that Stochastic Dominance is the *only* normative principle of decision theory. This

[25] See also Briggs (2015)

theory allows rampant incomparability between prospects. Combined with a suitable axiology, it satisfies all of the premises of theorem 7 *except* Simple Comparability, while upholding neither Positive nor Negative Fanaticism. (Note, incidentally, that it also does not satisfy Separability.) However, it must be noted that Tarsney's rejection of Fanaticism is half-hearted. While extremely risky gambles are not deemed *better* than safe options, neither are they deemed *worse*: accepting a fanatical wager and rejecting it are also incomparable prospects, in this theory.

4 | TAKING STOCK

One of the main routes to Fanaticism is via expectational total utilitarianism (what Bostrom called "standard utilitarianism"). One of the attractions of that package is that it seems to make for a very elegant moral universe. You can have clean principles like Separability and the Sure Thing Principle, and there are powerful theorems about how such principles constrain mathematical representations of betterness (see Broome, 2004). The theory does make some counterintuitive predictions, but these may be worth taking in stride, since the theory is so tidy and principled … in simple cases where there are only finitely many different states of nature, and there are at most finitely many people.

But as soon as you consider prospects with infinitely many states and unbounded populations, everything comes apart. In this general setting we find that expectational totalism is a radically incomplete theory (see Hájek & Nover, 2008; Lauwers, 2017). There is no way of extending expectational totalism to infinite cases with the same elegance that we are used to from finite cases. Elegant principles must be given up, including Separability and Reflection.

Expectational totalism does still seem to be a consistent live option, and with it Fanaticism. It satisfies modified and truncated versions of the original elegant principles. But the modifications and truncations seem like they might be telling us that there is something wrong about the underlying ideas. Whatever motivations there are for the modified and truncated principles seem to equally well motivate the sweeping clean versions. So even though the Fanatical position is still *there*, it no longer stands out as the bold, austere, and systematic ethical framework that it once seemed.

Furthermore, what Fanaticism has lost in elegance, anti-Fanatical theories have gained. For in fact, we *can* have sweeping, clean principles like Separability and Reflection, as long as we *give up* Fanaticism, along with some of the premises that got us there.

Here are three broad versions of this. One approach keeps both Reflection and Separability, while cutting our axiology down to size by giving up Compensation. A second keeps Reflection and Compensation, and gives up both Separability and Background Independence. Both of these two approaches can be represented using standard expected utility theory with bounded utilities; they only differ in the structure of their utility functions. (In the first case only, the utility function is additively separable.) A third elegant vision is Tarsney's theory, which keeps most of these principles (though not Separability) by giving up many comparisons of value.

All of these options are strange. I don't know which is true (if any), and I think it is premature to be confident in any of them.

The paradoxes of large values and small probabilities are deeply weird. But they aren't outlandish. In our *actual situation*, I take it that there are infinitely many live possibilities for what our universe is like, and we should assign significant probability to very good and very bad outcomes. Our own species might thrive for a very long time, and for all we know there may be many

others in the universe. These paradoxes are not just brainteasers that can be ignored when we are doing serious practical ethics—they raise difficult questions that we must answer, if we are to do as much good as we can.

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APPENDIX A: THEOREMS AND PROOFS

A.1 | Technical framework

There is a set of *states* equipped with a σ-algebra of *events*, and a set of *(finite) outcomes*. A *prospect* is a measurable function from states to outcomes. (We will focus on discrete prospects.) We fix in the background some probability measure P on states; we assume this is suitably rich (e.g., non-atomic). There is a relation $\succsim$ (*at least as good*) that holds between prospects; this is assumed to be transitive and reflexive. Strict betterness $\succ$ and indifference $\sim$ are defined in terms of $\succsim$

in the usual way. We will generally treat outcomes interchangeably with their corresponding constant prospects.

For convenience of exposition, we take as fixed some baseline "zero" outcome 0. Outcomes better than this are *good* or *gains*, and outcomes worse than this are *bad* or *losses*; outcomes exactly as good as the baseline are *neutral*. We take for granted that there is at least one good outcome. For an outcome x, we let $p * x$ stand for an arbitrary prospect with probability p of outcome x and probability $1 - p$ of the outcome 0.

(Positive) Fanaticism. For any probability $p > 0$ and any finite outcome $x \succ 0$, there is a finite outcome y such that $p * y \succ x$.

That is to say, any prospect which has probability p of outcome y and probability $1 - p$ of 0 is strictly better than the constant prospect which has outcome x in every state.

Negative Fanaticism. For any probability $p > 0$ and any finite outcome $x \prec 0$, there is a finite outcome y such that $p * y \prec x$.

There is also a set of *near outcomes* and a set of *far outcomes*. Each finite outcome has a *near* component and a *far* component. It will simplify our reasoning if we suppose that near and far outcomes are freely recombinable: for each near outcome x and far outcome y, there is a combined outcome $x \oplus y$ with those components. (This assumption could be weakened.) Without too much risk of confusion, we also use the notation 0 for the near and far components of the baseline outcome, so $0 = 0 \oplus 0$.

Near/far prospects are measurable functions from states to near/far outcomes. For a near prospect X and a far prospect B, we define the combined prospect statewise:

$$(X \oplus B)(s) = X(s) \oplus B(s)$$

If X and Y are near prospects, let $X \succ Y$ abbreviate $X \oplus 0 \succ Y \oplus 0$, and analogously for far prospects.

We will restate the principles that figure in the following theorems for easy reference.

Simple Separability. For any simple prospects X, Y, and B, $X \succ Y$ iff $X \oplus B \succ Y \oplus B$.

It is also useful to distinguish this special case:

Outcome Separability. For any near outcomes x and y and any far outcome b,

$$x \succ y \quad \text{iff} \quad x \oplus b \succ y \oplus b$$

For any far outcomes x and y and any near outcome b,

$$x \succ y \quad \text{iff} \quad b \oplus x \succ b \oplus y$$

We also must precisely state the *Compensation* principle that played a role in many arguments. This says that even if we zero out all of the near value, we can offset this by improving things enough in a distant galaxy.

(Positive) Compensation. For any near good x and far good y, there is a near good z such that $x \oplus y \sim z \oplus 0$, and there is a far good z' such that $x \oplus y \sim 0 \oplus z'$.

This amounts to a kind of unboundedness of moral value, in a sense that is not directly tied to risk. Say that a sequence of near outcomes $x_1, x_2, \ldots$ form an *arithmetic progression*, with difference z (which is a far good), iff

$$x_n \oplus z \sim x_{n+1} \oplus 0 \quad \text{for each } n$$

Positive Compensation implies that for any near good x_1 and any far good z, x_1 is the start of an infinite arithmetic progression with difference z. Furthermore, Outcome Separability implies that $x_1 \prec x_2 \prec \cdots$. We can say the same things about arithmetic progressions of far goods, where the difference is a near good.

A.2 | Distant space and time

Theorem 3. *Stochastic Dominance, Simple Separability, and Compensation together imply Fanaticism.*

We first prove a lemma.

Lemma 1. *Positive Compensation and Outcome Separability imply that, for any near good x and any probability $p > 0$, there is a near good y, a prospect $Y = p * y$, and a simple prospect B such that $x \oplus B$ is stochastically dominated by $Y \oplus B$.*

This lemma suffices for theorem 3: Stochastic Dominance ensures that $x \oplus B \prec Y \oplus B$, and then by Simple Separability $x \prec Y$, QED.

Proof of lemma 1. Assume $p < 1$. Consider a partition of $n + 1$ events: the first has probability p, and the rest each have probability $q = (1 - p)/n$, with n chosen to be large enough so that $q < p$.

By Positive Compensation, we can let $0, z_1, z_2, \ldots, z_n$ be an arithmetic progression of far goods with difference x: that is, for each k,

$$x \oplus z_k \sim 0 \oplus z_{k+1}$$

We can also choose a near good y such that

$$x \oplus z_n \sim y \oplus 0$$

By Outcome Separability,

$$x \oplus 0 \prec x \oplus z_1 \prec x \oplus z_2 \prec \cdots \prec x \oplus z_n \sim y \oplus 0$$

Then we construct prospects X, Y, and B as in table 6: X is sure to have outcome x, Y is a gamble with chance p of y, and B yields nothing if y pays off, and otherwise yields the result of a fair lottery between each of the outcomes $z_1, z_2, \ldots, z_n$.

It can be checked that $Y \oplus B$ stochastically dominates $x \oplus B$. □

A.3 | Separability against Fanaticism

Theorem 4. *Stochastic Dominance, Separability, and Compensation are jointly inconsistent.*

Proof. First a basic fact: for each outcome $x \oplus 0$, there is some strictly better outcome $y \oplus 0$ (and similarly for far outcomes). We have assumed that there is some good outcome; by Compensation some outcome $0 \oplus z$ is good. Then Compensation and Outcome Separability tell us that there is some $y \oplus 0 \sim x \oplus z \succ x \oplus 0$.

Now we recursively construct two sequences of finite goods as follows. For the base case, let $x_0 \oplus y_0 = 0 \oplus 0$. For the recursive step, for each n, we can find w_n and z_n such that

$$x_{n-1} \oplus 0 \prec z_n \oplus 0$$

$$0 \oplus y_{n-1} \prec 0 \oplus w_n$$

By Compensation we can then find x_n and y_n such that

$$w_n \oplus z_n \precsim 0 \oplus y_n$$

$$w_n \oplus z_n \precsim x_n \oplus 0$$

Next, we will use these outcomes to construct four prospects X, Y, W, Z as in table 7. Choose events $E_1, E_2, \ldots$ and $F_1, F_2, \ldots$, where $P(E_n) = P(F_n) = 2^{-(n+1)}$ (just as in Section 2).

By construction, $X \oplus Y$ is at least as good as $W \oplus Z$ in every state. But also, $W \oplus 0$ stochastically dominates $X \oplus 0$, and $0 \oplus Z$ stochastically dominates $0 \oplus Y$. So $X \oplus 0 \prec W \oplus 0$ and $0 \oplus Y \prec 0 \oplus Z$, and thus Separability tells us:

$$X \oplus Y \prec W \oplus Y \prec W \oplus Z \precsim X \oplus Y$$

This cannot be. □

Next we give the precise version of the result mentioned in Section 2.3: Stochastic Dominance, Separability, and two further conditions on the structure of outcomes are inconsistent with Fanaticism. The first condition says that the "near" and "far" domains are on a par with respect to potential value.

Symmetry. For any near outcome x, there is some far outcome y such that $x \oplus 0 \precsim 0 \oplus y$. Likewise, for any far outcome y, there is some near outcome x such that $0 \oplus y \precsim x \oplus 0$.

The second condition is that the orderings of near outcomes and far outcomes each have the structure of a *directed set*, which means that pairwise upper bounds exist.

Directed Set. For any near (far) outcomes x and y, there is some near (far) outcome z such that $x \precsim z$ and $y \precsim z$.

This condition immediately follows from the principle that any two outcomes are comparable. (Consider whichever of x and y is best.) But full comparability is much stronger than we need.

Theorem 8. *Stochastic Dominance, Separability, Symmetry, Directed Set, and Fanaticism are jointly inconsistent.*

Note that in the proof of theorem 4, it suffices that for each combined good $x \oplus y$, we can find some far good that is *at least* as good as $x \oplus y$—we don't need it to be exactly as good. So theorem 8 follows from the following lemma.

Lemma 2. *Stochastic Dominance, Separability, Symmetry, Directed Set, and Fanaticism together imply that for each good $x \oplus y$, there is some near good z such that $x \oplus y \precsim z \oplus 0$, and some far good z' such that $x \oplus y \precsim 0 \oplus z'$.*

Proof. Consider any near x and far y. By Fanaticism, there is some good $u \oplus v$ such that $x \oplus y \prec \frac{1}{2} * (u \oplus v)$. By Symmetry, there is also some near good w such that $0 \oplus v \precsim w \oplus 0$, and by Directed Set, there is some near good z such that $u \oplus 0 \precsim z \oplus 0$ and $0 \oplus v \precsim w \oplus 0 \precsim z \oplus 0$.

Write $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ for a prospect which has probability $1/2$ of outcome $a \oplus c$ and probability $1/2$ of outcome $b \oplus d$. Then we have

$$
\begin{aligned}
x \oplus y = \begin{pmatrix} x & x \\ y & y \end{pmatrix} &\prec \begin{pmatrix} u & 0 \\ v & 0 \end{pmatrix} \\
&\precsim \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix} && \text{by Stochastic Equivalence and Separability} \\
&\precsim \begin{pmatrix} z & z \\ 0 & 0 \end{pmatrix} = z \oplus 0 && \text{by Stochastic Dominance}
\end{aligned}
$$

The case of a far good goes similarly. □

Notice that this proof actually shows that these assumptions rule out Fanaticism even for the case where $p = 1/2$.

A.4 | Indology and Reflection

Now we will also consider *conditional* prospects: if X is a prospect and E is an event with positive probability, then $X|E$ is the restriction of X to E. We take the betterness relation to apply to conditional prospects as well. We also understand Stochastic Dominance as applying to this extended relation. Let $X \succsim_E Y$ mean that $X|E \succsim Y|E$.

In Section 3 we stated Reflection principles in terms of the answers to a question. In footnote 15 we introduced a more official model for this: a *regular partition* is a set $\mathcal{E}$ of mutually exclusive and jointly exhaustive events such that $P(E) > 0$ for each $E \in \mathcal{E}$.

Negative Reflection. For any prospects X and Y and any regular partition $\mathcal{E}$, if $X \not\succ_E Y$ for each $E \in \mathcal{E}$, then $X \not\succ Y$.

Negative Compensation. For any near good x and any far outcome y, there is a far outcome z such that $x \oplus z \sim 0 \oplus y$. For any near outcome x and any far good y, there is a near outcome w such that $w \oplus y \sim x \oplus 0$.

This lets us run compensation "downward": not only can we offset making nearby things worse by making far away things sufficiently better, but also we can offset making nearby things *better* by making far away things sufficiently *worse*.

Negative Compensation implies that we can also extend arithmetic progressions of outcomes *downward*: so Positive and Negative Compensation together tell us that for any pair of near (far) outcomes x_0, x_1 can be extended to an arithmetic progression ..., $x_{-2}, x_{-1}, x_0, x_1, x_2$, ... Outcome Separability implies that if $x_0 \prec x_1$ then each outcome in the sequence is better than those before it.

Theorem 5. *Stochastic Dominance, Negative Reflection, Background Independence, and Positive and Negative Compensation together imply Fanaticism.*

The theorem follows from the following lemma.

Lemma 3. *Positive and Negative Compensation and Outcome Separability together imply that for any good x and any probability $p > 0$, there is a prospect $Y = p * y$ and an independent (discrete) prospect B such that $x \oplus B$ is stochastically dominated by $Y \oplus B$.*

(The key difference from lemma 1 is that the background prospect B is now required to be *independent* of Y; but it can no longer be guaranteed to be a *simple* prospect.)

From lemma 3, we can apply the same reasoning as in Section 3: by Stochastic Dominance, $x \oplus B \prec Y \oplus B$; by Negative Reflection, B must have some possible outcome b such that $x \oplus b \prec Y \oplus b$; so by Background Independence $x \prec Y$.

Proof sketch for lemma 3. This is essentially the same as Tarsney's "Sufficiency Theorem" (2020), adapted to our more general setting, so we will not go into details. Here is the main idea.

Let x be a near good. Our Compensation principles tell us that there is an infinite arithmetic progression of far goods $\ldots, z_{-2}, z_{-1}, z_0, z_1, z_2 \ldots$, with difference x, where $z_0 = 0$. We let these be the possible outcomes of the background prospect B. We assign these outcomes probabilities that are sufficiently spread out. A *discrete Laplace distribution* will do:

$$\beta(n) = \alpha\, 2^{-|qn|}$$

where α is a normalization constant and $q < p/2$. The smaller the probability p is, the wider this distribution will be. Then we let y be a near good which is as good as some good enough far outcome z_N (where $N > 1/q$). It can be shown that if $Y = p * y$, and Y and B are independent, then $Y \oplus B$ stochastically dominates $x \oplus B$. □

A.5 | Reflection against Fanaticism

Theorem 6. *Stochastic Dominance and Negative Reflection together imply that Fanaticism is false.*

Proof. First, by Fanaticism we can construct a fast-growing sequence of outcomes: $x_1 = 0$, and for each $n > 1$, x_n is an outcome such that $2^{-n} * x_n \succ x_{n-1}$.

We flip two coins—Coin A and Coin B—until each of them has landed heads. Let A_n be the event where Coin A first comes up heads on the nth flip, and similarly for B_n. So

$$P(A_m B_n) = 2^{-(m+n)} \quad \text{for each } m, n \geq 1$$

Let X be a prospect that has outcome x_m in each event A_m. Then (by Stochastic Dominance) for each $n > 1$,

$$X \succ 2^{-n} * x_n \succ x_{n-1}$$

This also holds conditional on each event B_n (since X is independent of these events).

Let Y be a prospect that has outcome x_{n+1} in each event B_n. Then Y stochastically dominates X; so $Y \succ X$. But the events B_n form a regular partition, and conditional on each event B_n, Y is just as good as x_{n+1}, while X is strictly better than x_{n+1}. So Y is not better than X conditional on any B_n, which contradicts Negative Reflection. □

A.6 | The Sure Thing Principle and Fanaticism

The Sure Thing Principle. For any event E such that $P(E) > 0$, for any prospects X and Y such that $X \sim_{\neg E} Y$,

$$X \precsim Y \quad \text{iff} \quad X \precsim_E Y$$

We'll use the notation $(p * X,\ (1-p) * Y)$ for an arbitrary prospect such that the chance of any outcome x is

$$p \cdot P[X = x] + q \cdot P[Y = x]$$

We simplify $(p * X,\ (1-p) * 0)$ to $p * X$.

The Sure Thing Principle and Stochastic Equivalence together imply:

Independence. For any prospects X, Y, Z, and any probability $p > 0$,

$$X \precsim Y \quad \text{iff} \quad (p * X,\ (1-p) * Z) \precsim (p * Y,\ (1-p) * Z)$$

We also suppose:

Simple Comparability. For any good outcome x and bad outcome y,

$$\left(\frac{1}{2} * x,\ \frac{1}{2} * y\right) \succsim 0 \quad \text{or} \quad \left(\frac{1}{2} * x,\ \frac{1}{2} * y\right) \precsim 0$$

Theorem 7. *The Sure Thing Principle, Stochastic Equivalence, Background Independence, Positive and Negative Compensation, and Simple Comparability together imply that at least one of Positive Fanaticism or Negative Fanaticism is true.*

Proof. The proof has three steps.

Step 1. If Negative Fanaticism is false, there exists a bad outcome x such that for all y, $1/2 * y \not\prec x$.

If Negative Fanaticism is false, this means that there is a bad outcome x such that for some probability $p > 0$, for all bad outcomes y, $p * y \not\prec x$. Call a probability p *nice* iff it has the property that $p * y \not\prec x$ for all bad outcomes y. There are two cases to consider.

1. Some $p \geq 1/2$ is nice. Suppose y is a bad outcome such that $\frac{1}{2} * y \prec x$. By Independence,

$$p * y \precsim \frac{1}{2} * y \prec x$$

This contradicts the assumption that p is nice. So there is no such y.

2. No $p \geq 1/2$ is nice. In that case there is some nice $p < 1/2$ such that $2p$ is not nice: that is, there is some bad outcome x' such that $2p * x' \prec x$. Now suppose that y is a bad outcome such that $\frac{1}{2} * y \prec x'$. By Independence,

$$p * y \precsim 2p * x' \prec x$$

This contradicts the assumption that p is nice.

Step 2. For any good outcome x, there is an arithmetic progression $x^-, 0, x^+$ such that $x \precsim x^+$, and

$$\left(\frac{1}{2} * x^-,\ \frac{1}{2} * x^+\right) \succsim 0$$

Let y^- be a bad outcome as given by Step 1; without loss of generality we can suppose y^- is a *far* outcome, by Negative Compensation. Positive Compensation then ensures that there is an arithmetic progression $y^-, 0, y^+$ with difference d. For any outcome z, we can choose a far outcome z' such that $z \sim d \oplus z'$. The property from Step 1 tells us:

$$\left(\frac{1}{2} * z',\ \frac{1}{2} * 0\right) \not\succ y^-$$

By Background Independence, we can add d to each outcome, which yields:

$$\left(\frac{1}{2} * z,\ \frac{1}{2} * y^+\right) \not\succ 0$$

By Simple Comparability, then, for any bad outcome z,

$$\left(\frac{1}{2} * z,\ \frac{1}{2} * y^+\right) \succsim 0$$

Now let $x^+ = x \oplus y^-$. This is better than either x or y^+, since they are each good (using Outcome Separability). And there is an arithmetic progression $x^-, 0, x^+$, such that (by Independence),

$$\left(\frac{1}{2} * x^-,\ \frac{1}{2} * x^+\right) \succsim \left(\frac{1}{2} * x^-,\ \frac{1}{2} * y^+\right) \succsim 0$$

Step 3. Deduce Positive Fanaticism.

We can show by induction that for each $n > 0$, there is some (near) good outcome y such that

$$\frac{1}{n} * y \gtrsim x^+$$

The base case is clear. For the inductive step, suppose this holds for n. By Independence,

$$\left(\frac{1}{2} * \frac{1}{n} * y,\ \frac{1}{2} * x^-\right) \succsim \left(\frac{1}{2} * x^+,\ \frac{1}{2} * x^-\right) \gtrsim 0$$

The left hand side can be rewritten:

$$\frac{n+1}{2n} * \left(\frac{1}{n+1} * y,\ \frac{n}{n+1} * x^-\right)$$

(and 0 can be rewritten as $\frac{n+1}{2n} * 0$), so by Independence,

$$\left(\frac{1}{n+1} * y,\ \frac{n}{n+1} * x^-\right) \gtrsim 0$$

Background Independence lets us add z to each outcome.

$$\left(\frac{1}{n+1} * (y \oplus z),\ \frac{n}{n+1} * 0\right) \gtrsim x^+$$

Finally, by Compensation, there is a near outcome y' such that $y' \oplus 0 \sim y \oplus z$, and we have

$$\frac{1}{n+1} * y' \gtrsim x^+$$

completing the induction.

Finally, for any probability $p > 0$, we can choose n such that $1/n < p$. For some good y,

$$p * y \succ \frac{1}{n} * y \gtrsim x^+ \gtrsim x$$

□