Aristotle on Principles as Elements

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Draft, September 2016

Abstract: In his discussion of the four causes, Aristotle claims that 'the hypotheses are material causes of the conclusion' (*Physics* 2.3, *Metaphysics* Δ 2). This claim has puzzled commentators since antiquity. It is usually taken to mean that the premises of any deduction are material causes of the conclusion. By contrast, I argue that the claim does not apply to deductions in general but only to scientific demonstrations. In Aristotle's view, the theorems of a given science are composites composed of the indemonstrable premises from which they are demonstrated. Accordingly, these premises are elements, and hence material causes, of the theorems. Given this, Aristotle's claim can be shown to be well-motivated and illuminating.

1. Hypotheses are material causes of the conclusion

In *Physics* 2.3 and its doublet in *Metaphysics* Δ 2, Aristotle distinguishes the so-called four causes. The first of them is characterized as 'that from which as a constituent something comes to be'.[1] Since antiquity, this type of cause is known as the 'material cause'.[2] Aristotle illustrates it by a series of examples as follows:

[1] τὸ ἐξ οὗ γίγνεταί τι ἐνυπάρχοντος, *Phys.* 2.3 194b24 (= *Metaph.* Δ 2 1013a24–5).

[2] The ancient commentators refer to it as ὑλικὸν αἴτιον (Alex. Aphr. *in Metaph.* 349.2, 351.5, 353.20, Philop. *in Phys.* 243.21, 243.30, 246.25–247.11, 249.6, Simpl. *in Phys.* 314.27, 319.20, 320.13, Asclep. *in Metaph.* 305.22–4, 306.9–14). Moravcsik argues that this type of cause should rather be called 'constitutive

> Letters are causes of syllables, matter of artefacts, fire and the like of bodies, the parts of the whole, and the hypotheses of the conclusion, as that from which.
> τὰ μὲν γὰρ στοιχεῖα τῶν συλλαβῶν καὶ ἡ ὕλη τῶν σκευαστῶν καὶ τὸ πῦρ καὶ τὰ τοιαῦτα τῶν σωμάτων καὶ τὰ μέρη τοῦ ὅλου καὶ αἱ ὑποθέσεις τοῦ συμπεράσματος ὡς τὸ ἐξ οὗ αἴτιά ἐστιν (*Phys.* 2.3 195a16–19 = *Metaph.* Δ 2 1013b17–21)

In this passage, Aristotle lists five examples of material causes.[3] While the first four examples are relatively straightforward, the last one is puzzling and has proved difficult to understand. Aristotle does not explain what he means when he writes that the hypotheses are material causes of the conclusion. Commentators have, of course, noted that a conclusion is typically inferred from premises (προτάσεις) in a deduction (συλλογισμός). Accordingly, it is often thought that the term 'hypotheses' (ὑποθέσεις) in Aristotle's example is used interchangeably with 'premises', referring to the premises of any

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factor', since not all of its instances can be regarded as matter (ὕλη); see Moravcsik 1974: 7, 1975: 627, 1991: 35 and 44. I will use 'material cause' as a neutral label, without implying that every material cause of an item is matter of that item.

[3] It is widely agreed that each of these examples is meant to be an instance of the material cause (Alex. Aphr. *in Metaph.* 351.3–15, Themistius *in Phys.* 45.12–18, Philop. *in Phys.* 246.24–247.11, Simpl. *in Phys.* 319.16–26, Pacius 1596: 448, Zabarella 1601: ii.54, Bonitz 1849: 223, Maier 1900: 175 n. 2, Charlton 1970: 100). Ross suggests that ὡς τὸ ἐξ οὗ αἴτια at 195a19 (= 1013b20–1) is used in a wide sense covering both material and formal causes (Ross 1924: i 293, 1936: 512). However, the reasons adduced by him for this reading are not convincing (see Philop. *in Phys*. 247.7–13, Pacius 1596: 448, Wagner 1995: 464). Even if Ross's reading is right, none of the five examples listed in the passage is an instance of the formal cause.

deduction.[4] Thus, Aristotle is usually taken to claim that the premises of any deduction are material causes of its conclusion.[5]

While this reading of Aristotle's claim is widely accepted, the claim itself has remained obscure. It is not clear in what sense the premises of any deduction can be regarded as material causes of its conclusion. Consider, for example, a deduction inferring a true conclusion from false premises, such as the following:

Every human is a stone.
Every stone is an animal.
Therefore, every human is an animal.

Although the two premises are obviously false, Aristotle is clear that this argument is a perfectly valid deduction (*APr.* 2.2 53b26–35). Yet it is not easy to see on what grounds the two premises might count as material causes of the conclusion 'Every human is an

[4] Simpl. *in Phys.* 319.33–320.11, Pacius 1596: 448, Waitz 1844: 428, Bonitz 1849: 219, Sigwart 1871: 1, Hamelin 1907: 92, Thiel 1919: 15–16, 1920: 1, Ross 1924: i 292–3, 1936: 352 and 512, Mure 1928: ad 94a21–2, Balme 1972: 83, Mignucci 1975: 39, Barnes 1990: 40, 2016: 139, Goldin 1996: 47 n. 10 and 54 n. 26, Crivelli 2011: 122–3, Ebrey 2015: 193 n. 22. Accordingly, ὑποθέσεις in this passage is often rendered as 'premises'; Alex. Aphr. *in Metaph.* 351.8, Themistius *in Phys.* 45.18, Philop. *in Phys.* 247.5–6, Zabarella 1601: ii.54, Charlton 1970: 100, Barnes 1994: 226, Byrne 2001: 88, Bostock 2006: 84 n. 14.

[5] Philop. *in Phys.* 247.5–6, Pacius 1596: 448, Zabarella 1601: ii.54, Maier 1900: 175 and 223, Ross 1924: i 292–3, 1936: 512, Mure 1928: ad 94a21–2, Charlton 1970: 100, Happ 1971: 798 n. 598, Balme 1972: 83, Wieland 1992: 211, Detel 1993: i 303 and ii 701, Barnes 1994: 226, 2016: 139, Bostock 2006: 84 n. 14, Winter 2011: 73 n. 22, Thom 2013: 135, Ebrey 2015: 193 n. 22, Castagnoli 2016: 16.

animal'. More generally, the traditional reading implies that, for any arbitrary term C, the premises 'Every human is C' and 'Every C is an animal' are material causes of that conclusion. Perhaps for this reason, Aristotle's claim has been deemed problematic since antiquity. Thus, when Alexander comments on the claim, he feels compelled to modify it as follows:

> The premises are causes of the whole deduction by their combination, for they are causes of the conclusion not as matter, but as a productive cause; and in the whole deduction the premises are like matter, but the conclusion like form. (Alexander *in Metaph.* 351.12–15)

Alexander denies that the premises of a deduction are material causes of its conclusion. Instead, he takes them to be efficient causes of the conclusion. In addition, he suggests that the premises are material causes of the entire deduction, while the conclusion is its formal cause.[6] This latter suggestion was acccepted by Philoponus, and there are echoes of it in later authors such as Kant.[7] Nevertheless, it seems clear that this is not what Aristotle had in mind; for he states that the hypotheses are material causes not of the

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[6] See also the report of Alexander's view given by Simplicius, *in Phys.* 320.1–10.

[7] Philop. *in APr.* 6.10–14, 32.31–33.2, 387.9–11, *in Phys.* 166.14. Similarly, Kant writes in §59 of the *Jäsche Logic* that 'the matter of inferences of reason consists in the antecedent propositions or premises, the form in the conclusion insofar as it contains the *consequentia*' (Kant 1923: 121; cf. Longuenesse 2005: 226); see also Meier 1752: §359.

whole deduction but of the conclusion.[8] Nor does Aristotle state that the premises are efficient causes of the conclusion.[9]

Thomas Aquinas suggests that the premises of a deduction can be viewed as material causes of the conclusion on the grounds that they contain all the terms that occur in the conclusion (i.e., the major and minor terms).[10] Again, it is unlikely that this is what Aristotle had in mind. Aristotle takes material causes to be constituents (ἐνυπάρχοντα) of the things of which they are material causes; and it would be misleading to say that the premises are constituents of the conclusion when it is not the premises but only some of the terms contained in them that are constituents of the conclusion. This would be especially questionable in the case of deductions from more than two premises, which contain a plurality of middle terms that do not occur in the conclusion.

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[8] Barnes 2016: 139. Pacius (1596: 448) argues that Aristotle uses συμπέρασμα at 195a18–19 (= 1013b20) to refer not to the conclusion of a deduction but to the entire deduction. However, there is no textual support for this reading. While συλλογισμός is sometimes used by Aristotle to refer to the conclusion of a deduction (Bonitz 1870: 712a9–11), there is no evidence that he uses συμπέρασμα to refer to entire deductions (cf. Bonitz 1870: 717a38–42). Pacius suggests that συμπέρασμα is used at *Phys*. 2.7 198b8 to refer to a deduction as a whole; but this is not correct (see n. 20).

[9] It is unlikely that the phrase ὡς τὸ ἐξ οὗ at 195a19 covers efficient causes; for Aristotle discusses efficient causes at 195a21–3, and none of the other four examples listed at 195a16–18 is an instance of the efficient cause.

[10] Aquinas *in Metaph.* 778; similarly, Zabarella 1601: ii.54.

Relatively little progress has been made over the centuries since Aquinas. Commentators often dismiss Aristotle's claim, following David Bostock's advice to 'set aside, as irrelevant to Aristotle's main thought, the odd suggestion that in an argument the premisses are the 'material cause' of the conclusion'.[11]

The purpose of this paper is to vindicate Aristotle's claim by offering a new interpretation of it and situating it in the broader context of his writings. I argue that the claim applies not to the premises of any deductions, but specifically to the premises of scientific demonstrations (ἀποδείξεις). It is only the latter that Aristotle regards as material causes of the conclusion. So understood, Aristotle's claim turns out to be well-motivated and illuminating.

I begin by arguing that the 'hypotheses' referred to in Aristotle's claim are not premises of deductions in general, but indemonstrable premises of demonstrations (Section 2). As Aristotle explains in *Posterior Analytics* 1.23, these premises can be viewed as elements (στοιχεῖα) of the theorems demonstrated from them (Section 3). As such, they are material causes of the theorems (Section 4). This is in accordance with the fact that mathematicians associated with the Academy referred to the elementary propositions of geometry as 'elements'. Moreover, it fits well with ancient conceptions of geometrical analysis, according to which theorems are analyzed – or decomposed – into the principles from which they are demonstrated (Section 5).

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[11] Bostock 2006: 84 n. 14; although there are helpful remarks on Aristotle's claim, e.g., in Charlton 1970: 100 and Crubellier 2008: 126 (on which see n. 101).

2. Hypotheses are principles of demonstrations

It is important to note that Aristotle's claim in *Physics* 2.3 refers not to 'premises' (προτάσεις) but to 'hypotheses' (ὑποθέσεις). While Aristotle uses the term 'hypothesis' in a wide variety of ways, he never uses it to mean 'premise'. It is true that Aristotle applies the term to propositions which serve as premises in certain types of deduction. Thus, for example, he applies it to the assumption for *reductio* in deductions by *reductio ad absurdum*.[12] But this does not mean that 'hypothesis' is used as a general term applying to any premise of any deduction. Aristotle never defines such a wide sense of the term, and when he wishes to refer to the premises of deductions in general he speaks of 'premises' and 'what has been laid down' (τὰ κείμενα), but not 'hypotheses'.[13]

[12] *APr.* 1.15 34b29, 2.11 61b32 and 62a4, 2.13 62b12 and 62b20, 2.14 *passim*, 2.17 *passim*; cf. von Fritz 1955: 37. In addition, Aristotle uses 'hypothesis' in the context of modal proofs to refer to the assumption that something known to be possible actually is the case (*APr.* 1.15 34a26, *Phys.* 7.1 243a30); cf. Rosen & Malink 2012: 193.

[13] Goldin (1996: 54 n. 26) suggests that 'hypothesis' is used to mean 'premise' at *APr.* 1.23 40b25–9, 41a23–7, 1.44 50a16–b4, *Top.* 3.6 119b35–9. These passages deal with what Aristotle calls 'deductions from a hypothesis' (συλλογισμοί ἐξ ὑποθέσεως). The hypothesis in virtue of which these deductions are so called is a convention or agreement that, in Aristotle's view, does not count as a premise of the deduction (Striker 1979: 42–3, 2009: 236, Bobzien 2002: 371, Crivelli 2011: 142–9). Thus, 'hypothesis' is not used in these passages as a general term applying to the premises of any deduction (Crivelli 2011: 122–3). Similarly, there is no evidence that 'hypothesis' means 'premise' at *APost.* 1.10 76b35–9 (*contra* Goldin 1996: 54 n. 26, McKirahan 1992: 47). Instead, this occurrence of 'hypothesis' may be understood in the sense defined at *APost.* 1.2 72a19–20 or 1.10 76b27–30; cf. Crivelli 2011: 123 n. 58.

In the *Posterior Analytics*, on the other hand, Aristotle defines a technical sense of 'hypothesis' in which the term refers to a special kind of principle of demonstration (ἀρχή ἀποδείξεως, 1.2 72a7). A demonstration is a deduction that is capable of conferring scientific knowledge (ἐπιστήμη, 71b18–19). In order for a demonstration to have this capacity, it must satisfy a number of conditions that distinguish it from deductions in general. For example, the premises of a demonstration must be true, prior to, and explanatory of the conclusion (71b19–33). Hypotheses are premises of demonstrations that satisfy these conditions.[14] Being principles of demonstration, they are indemonstrable (72a14–15). More specifically, Aristotle defines a hypothesis as a principle

> which assumes either kind of assertion, I mean that something is or that something is not.[15] (*APost.* 1.2 72a19–20)

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[14] This is not to say that all principles of a science are premises of demonstrations; for example, definitions are arguably not assertions and therefore not capable of serving as premises (*APost.* 1.2 72a21–4, 1.10 76b35–8; see Philop. *in APost.* 131.28–132. 23, Pacius 1597a: 418, Robinson 1953: 101–2, Hintikka 1972: 66–9). Nevertheless, it is clear that Aristotle takes hypotheses to be premises of demonstrations (Hintikka 1972: 66–7). His characterization of hypotheses at 72a19 closely resembles his characterization of premises at 72a8–9. Moreover, Aristotle states that hypotheses are 'among the premises' (*APost.* 1.10 76b36), and describes them as assertions 'such that, if they are, then by their being so the conclusion comes about' (76b38–9). The latter phrase recalls the characterization of premises of deductions at *APr.* 1.1 24b19–20.

[15] ἡ μὲν ὁποτερονοῦν τῶν μορίων τῆς ἀποφάνσεως λαμβάνουσα, οἷον λέγω τὸ εἶναί τι ἢ τὸ μὴ εἶναί τι, ὑπόθεσις. This is the text printed by Bekker (1831) and Waitz (1846), whereas Ross (1949) reads

Thus, a hypothesis is an affirmative or negative assertion that has the status of an indemonstrable premise in a given science. For example, the assertion 'There is a unit' is a hypothesis of arithmetic (72a18–24). For our purposes it is not necessary to enter into a discussion of the precise nature of these hypotheses and their relationship to the other kinds of demonstrative principle identified by Aristotle in the *Posterior Analytics*.[16] What is important is that hypotheses are defined by him as indemonstrable premises of demonstrations.

ἀντιφάσεως instead of ἀποφάνσεως. The former reading is supported by 72a8–9. Nevertheless, both readings yield essentially the same result. The phrase τῶν μορίων τῆς ἀποφάνσεως refers to the two kinds of simple assertion, i.e., affirmation and denial (cf. *Int.* 5–6 17a8–9, 17a23–6; for the use of μόριον in the sense of 'kind' or 'species', see *Metaph.* Δ 25 1023b18–19 and Bonitz 1870: 473b59–60). Similarly, τῶν μορίων τῆς ἀντιφάσεως refers to the two members of a contradictory pair, i.e., affirmation and denial (cf. *Int.* 6 17a31–7).

[16] For example, it is debated whether the hypotheses introduced in *APost.* 1.2 are exclusively existential assertions, such as 'There is a unit'. Some commentators think that the answer is yes (Cornford 1932: 41, Lee 1935: 114 and 117, Mansion 1946: 153, Heath 1949: 53–5, Ross 1949: 55–7 and 508, von Fritz 1955: 38–40, Landor 1981: 311–13, McKirahan 1992: 41–4, Goldin 1996: 41–61). On the other hand, there is good reason to think that these hypotheses include non-existential assertions as well as existential ones (Philop. *in APost* 35.2–19, Pacius 1597a: 418, Robinson 1953: 101–3, Hintikka 1972: 66–9, Mignucci 1975: 36–9, Leszl 1981: 309–10, Harari 2004: 40–6). This view is supported by passages in which 'hypothesis' refers to non-existential demonstrative principles (e.g., *APost.* 1.19 81b14–15, *Phys.* 8.3 253b2–6; see von Fritz 1955: 38, Landor 1981: 311–14, Barnes 1994: 100, Harari 2004: 43–4).

This use of 'hypothesis' is not limited to the *Posterior Analytics* but is also common in Aristotle's non-logical writings, such as the *Eudemian Ethics* and *Physics*:[17]

> Just as in the theoretical sciences, the hypotheses are principles (ἀρχαί), so in the productive sciences the end is a principle and hypothesis. (*EE* 2.11 1227b28–30)

> Just as in arguments about mathematics objections concerning the principles (τῶν ἀρχῶν) do not affect the mathematician, . . . so, too, objections concerning the point just mentioned do not affect the physicist; for it is a hypothesis that nature is a principle of motion. (*Phys.* 8.3 253b2–6)

The same use of 'hypothesis' occurs in *Metaphysics* Δ 1:

> That from which a thing can first be known is called a principle of that thing, as for instance the hypotheses of demonstrations (τῶν ἀποδείξεων αἱ ὑποθέσεις). (*Metaph.* Δ 1 1013a14–16)

It is sometimes thought that the occurrence of 'hypothesis' in this passage does not have the sense defined in the *Posterior Analytics*, but a more general sense in which it applies

[17] See also *NE* 7.8 1151a17, *EE* 2.6 1222b28, 2.10 1227a7–11 (cf. Heath 1949: 278–80). For more examples, see Crivelli 2011: 123 n. 85.

to the premises of any deduction.[18] There is, however, no textual support for this reading. On the contrary, the fact that Aristotle speaks of 'hypotheses of demonstrations' suggests that he takes the hypotheses in question to be principles of demonstration just like in the *Posterior Analytics*. It is therefore preferable to follow Alexander and others in taking 'hypothesis' in this passage to refer to indemonstrable premises of demonstrations.[19]

Given that 'hypothesis' is used in this sense in *Metaphysics* Δ 1, it is natural to suppose that it is used in the same sense in Δ 2, when Aristotle claims that hypotheses are material causes of the conclusion. Moreover, Aristotle uses the term 'premises' (προτάσεις) in *Physics* 2.7 to refer to the premises of deductions in general.[20] Thus, he

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[18] Bonitz 1849: 219, Sigwart 1871: 1, Hamelin 1907: 92, Thiel 1919: 15–16, 1920: 1, Ross 1924: i 291, Goldin 1996: 47 n. 7 and 54 n. 26.

[19] Alex. Aphr. *in Metaph.* 346.24–32, Aquinas *in Metaph.* 759, Irwin 1988: 3, McKirahan 1992: 227, Crivelli 2011: 123 n. 58.

[20] In *Physics* 2.7, Aristotle describes material causes by the phrase 'if so and so is to be, as the conclusion from the premises' (εἰ μέλλει τοδὶ ἔσεσθαι, ὥσπερ ἐκ τῶν προτάσεων τὸ συμπέρασμα, 198b7–8; cf. Philop. *in Phys.* 305.21–5, Simpl. *in Phys.* 368.23–6). It is sometimes thought that this phrase appeals to Aristotle's claim at 2.3 195a18–19 that hypotheses are material causes of the conclusion (Pacius 1596: 474, Ross 1936: 528, Charlton 1970: 113, Sorabji 1980: 51 n. 24). However, this is not correct. As Malcolm Schofield (1991: 37) has shown, 198b7–8 pertains to the discussion of 'hypothetical necessity' at *Phys.* 2.9 200a11–30 and *PA* 1.1 639b21–640a6. In these passages, material causes are described as hypothetically necessary for the achievement of certain ends; for example, in order for there to be a house it is necessary for there to be bricks and stones (*Phys.* 2.9 200a24–30, *PA* 1.1 639b24–7). Aristotle compares this type of necessity to *necessitas consequentia*, i.e., the type of necessity with which the conclusion of a deduction follows from the premises (198b5–8). The point of 198b7–8 is that if X is to be then necessarily X's material cause must be,

would be able to use the same term in *Physics* 2.3 if he wanted to make a general claim about the premises of any deduction. Again, this suggests that Aristotle does not intend such a general claim but a more limited claim to the effect that, in every demonstration, the indemonstrable premises are material causes of the conclusion.

If this is correct, the claim does not apply to deductions such as the one considered above, in which a true conclusion is inferred from false premises. Nor does the claim apply to deductions in which the premises are true but not prior to and explanatory of the conclusion, such as the following:

> Whatever does not twinkle is near.
> The planets do not twinkle.
> Therefore, the planets are near.

This deduction fails to be a demonstration because the premises do not indicate the reason why (τὸ διότι) the conclusion holds; for it is not because the planets do not

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just like if the premises of a deduction are the case then necessarily its conclusion must be the case (Schofield 1991: 37; similarly, Aquinas *in Phys.* 248). Aristotle does not here view the premises of a deduction as material causes of the conclusion. Instead, he compares material causes to the conclusions of deductions (although he does not take these conclusions to *be* material causes). Since *necessitas consequentiae* is exhibited by any deduction, this comparison is not restricted to demonstrations. Accordingly, Aristotle speaks of 'premises' rather than 'hypotheses' at 198b7–8, indicating that he is referring to premises of deductions in general. Thus, Aristotle's point at 198b7–8 is independent from the one at 195a18–19. Accordingly, I do not agree with the view that at 200a15–30 Aristotle contradicts and corrects what he said at 195a18–19 (Ross 1949: 639, Dancy 1978: 376).

twinkle that they are near, but because they are near they do not twinkle (*APost.* 1.13 78a36–8).

By thus limiting the scope of Aristotle's claim to indemonstrable premises of demonstrations, the claim becomes weaker and hence more defensible than on the traditional interpretation. Nevertheless, we are left with the question as to why Aristotle chose to regard these premises as material causes of the conclusion. Linguistically, 'hypothesis' (ὑπόθεσις) is closely related to one of the terms Aristotle uses to describe material causes: namely, 'underlying subject' (ὑποκείμενον, *Phys*. 2.3 195a20 = *Metaph.* Δ 2 1013b21). But this alone hardly suffices as an explanation. Thus, Richard McKirahan writes that 'even if Aristotle did think that principles [of demonstrations] are the material cause of conclusions, it is unclear why he thought so or what he thought that identifying them in that way would achieve. Explicating principles in terms of the material cause is a dead end.'[21] In what follows, I argue that Aristotle's claim is not a dead end, but part of a well-motivated view of scientific demonstration that is developed elsewhere in Aristotle's writings. To this end, I turn to *Posterior Analytics* 1.23, where Aristotle explains how indemonstrable premises of demonstrations can be viewed as elements (στοιχεῖα) of the theorems demonstrated from them.

[21] McKirahan 1992: 228.

3. Indemonstrable premises are elements of theorems (*Posterior Analytics* 1.23)

Scientific knowledge, for Aristotle, is essentially tied to demonstration (ἀπόδειξις). In a demonstration, a proposition which serves as the conclusion is deduced from two or more propositions which serve as premises. Every proposition that falls under the purview of a given science either is or is not demonstrable within that science. If it is demonstrable, it is a theorem of the science; otherwise, it is a principle. In order to have scientific knowledge of a theorem, the scientist must possess a demonstration of it.[22]

In the *Analytics*, demonstrations take the form of deductions in the three syllogistic figures. For example, a universal affirmative proposition AaB ('A holds of all B') is demonstrated by means of a first–figure deduction in Barbara:

$$\frac{\text{AaC} \qquad \text{CaB}}{\text{AaB}}$$

Each of the two premises, AaC and CaB, either is or is not demonstrable. If it is demonstrable, then in order to have scientific knowledge of AaB, the demonstrator must have scientific knowledge of this premise through yet another demonstration.[23] This

[22] *APost.* 1.2 71b28–9, 1.22 83b34–5; cf. McKirahan 1992: 164.

[23] See *APost.* 1.3 72b20–2 and 1.22 83b34–8; cf. Philop. *in APost.* 254.24–255.25.

demonstration will again be of the form Barbara, since there is no other way to deduce a universal affirmative proposition in Aristotle's syllogistic theory:[24]

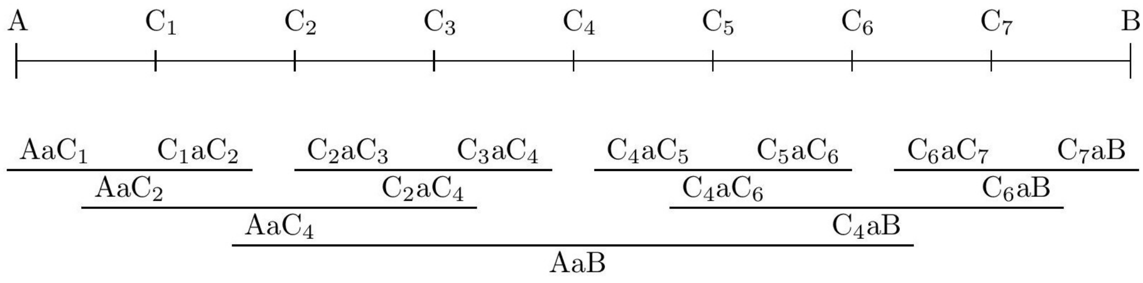
$$\dfrac{\mathrm{AaC} \qquad \dfrac{\mathrm{CaD} \qquad \mathrm{DaB}}{\mathrm{CaB}}}{\mathrm{AaB}}$$

The same argument applies to the new premises, CaD and DaB, and so on. Thus, the scientist faces a regress of demonstrations. However, Aristotle denies that the regress goes on to infinity. In *Posterior Analytics* 1.19–22, he provides an elaborate argument to the effect that every regress of demonstrations ultimately terminates in indemonstrable premises.[25] In the framework of Aristotle's syllogistic theory, this means that, if a proposition AaB is demonstrable, then there are finitely many middle terms that can be used to demonstrate it from indemonstrable premises. For example, if there are seven middle terms, the demonstration can be represented by a deduction tree such as the following:[26]

[24] See *APr.* 1.26 42b32–3. In *Posterior Analytics* 1.19–23, Aristotle does not discuss deductions by *reductio ad absurdum*. This is, in part, because he holds that *reductio* is dispensable in the assertoric syllogistic: given the fourteen syllogistic moods established in *Prior Analytics* 1.4–6, every conclusion that is deducible from given premises by means of *reductio* is also deducible from them without *reductio* (*APr.* 1.29 45a23–b11, 2.14 62b38–63b21; see Ross 1949: 454–6). In the absence of *reductio*, the only way to deduce an a-conclusion is by Barbara.

[25] See *APost.* 1.22 84a29–b2; cf. 1.3 72b18–22.

[26] See Lear 1980: 22–4.



The premises at the top of this tree are indemonstrable (AaC_1, C_iaC_{i+1}, C_7aB). They are principles of the science under consideration, and, more specifically, as Aristotle points out in chapter 1.19, hypotheses.[27]

It is noteworthy that, throughout *Posterior Analytics* 1.19–23, Aristotle uses spatial terminology to describe complex demonstrations such as the one just given. For example, he refers to the propositions that occur in the demonstration as ‘intervals’ (διαστήματα).[28] In doing so, he compares syllogistic propositions such as AaB to one-dimensional regions of space bounded by two points, with the terms A and B corresponding to the two endpoints. More concretely, Aristotle likens syllogistic propositions to musical intervals (διαστήματα), viewed as one-dimensional regions of musical space bounded by two pitches.[29] If an interval is indemonstrable (i.e., if there is

[27] *APost.* 1.19 81b14–15. I take ‘hypothesis’ here to have the sense defined at 1.2 72a19–20; see Crivelli 2011: 123 n. 58.

[28] *APost.* 1.21 82b7–8, 1.22 84a35, 1.23 84b14. Similarly, *APr.* 1.4 26b21, 1.15 35a12, 35a31, 1.18 38a4, 1.25 42b9–10, 2.2 53b20.

[29] He refers to musical intervals at 1.23 84b33–85a1. For the musical background of the use of διάστημα in the *Analytics*, see Smith 1978: 202–6. Aristotle’s pupil Aristoxenus characterizes a musical interval (διάστημα) as a space (τόπος) bounded by two musical pitches (*El. Harm.* 21.1–4). Thus, musical intervals

no middle term through which it can be demonstrated), Aristotle calls it ‘immediate’ (ἄμεσον).[30] Alternatively, he calls such intervals ‘indivisible’ (ἀδιαίρετον).[31] If AaB is indivisible in this sense, then A is said to hold of B atomically (ἀτόμως).[32] Accordingly, an indemonstrable interval is called ‘atomic’ (ἄτομον).[33] In the above tree, each of the premises at the top is atomic. On the other hand, if an interval is demonstrable, it is called ‘divisible’ (διαιρετόν).[34] In the above tree, the conclusion AaB is a divisible interval in which the terms A and B are separated by eight atomic intervals.

When a scientist constructs a complex demonstration from the bottom up, starting with the conclusion AaB and supplying new middle terms until every branch terminates in an indemonstrable premise, each middle term marks a division of the interval between A and B. By this insertion of middle terms between A and B, ‘the middle will always be densified (πυκνοῦται), until the [intervals] become indivisible and single’

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are special cases of intervals in general, thought of as one-dimensional regions of space between two boundaries (Arist. Quint. *De Mus.* 1.7 1–4).

[30] *APost.* 1.21 82b7, 1.22 84a35, 1.23 84b14, 84b22, 84b36–7, 85a1. For this use of ‘immediate’, see also 1.2 71b21, 72a7–8, 1.3 72b19, 1.14 79a31, 1.33 89a14, 2.8 93a36, 2.19 99b21–22; cf. McKirahan 1992: 25 and 276 n. 26.

[31] *APost.* 1.22 84a35, 1.23 84b35.

[32] *APost.* 1.15 79a33–4, 79b13, 79b21–2, 1.16 79b30, 80a3, 80a12, 1.17 80b17.

[33] *APost.* 1.16 80b16, 2.18 99b7 (on the latter passage, see Mure 1928: *ad loc.*, Ross 1949: 673, Tredennick 1960: 255, McKirahan 1992: 179–80, Barnes 1994: 257).

[34] *APost.* 1.22 84a35.

(1.23 84b34–5).[35] Aristotle's use of the verb 'densify' in this context derives from analogous uses in musical theory.[36] Densifications (πυκνώματα) are mentioned by Plato in connection with attempts by empirical harmonicists to identify a smallest musical interval of which larger intervals can be treated as multiples.[37] Similarly, Aristotle's student Aristoxenus attributes to these harmonicists a method of 'densifying diagrams', whereby musical intervals are represented diagrammatically as multiples of smallest intervals (i.e., quarter-tones).[38] While Aristoxenus does not describe these diagrams in any detail, it is likely that they took the form of a straight line marked off at equal distances representing the successive smallest intervals.[39] Larger musical intervals are then represented as measurable distances in geometrical space.

In *Posterior Analytics* 1.23, Aristotle compares the indemonstrable premises of demonstrations to the smallest intervals posited in musical theory: just like a smallest

[35] ἀεὶ τὸ μέσον πυκνοῦται, ἕως ἀδιαίρετα γένηται καὶ ἕν. In this passage, τὸ μέσον is not a middle term but the interval extending from the major to the minor term; see Mure 1928: *ad loc.*

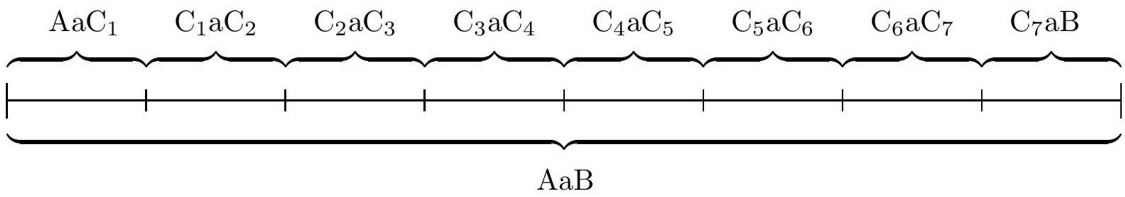
[36] The same is true for the occurrence of καταπυκνοῦται at *APost.* 1.14 79a30; see Einarson 1936: 158. I am grateful to Stephen Menn for drawing my attention to this point.

[37] *Resp.* 7 531a; cf. Barker 1989: 55–6 n. 3, 2007: 23–4 and 34.

[38] Aristox. *El. Harm.* 36.1–5; see also 12.8–15, 66.3–5. Cf. Monro 1894: 52–3, Barker 1989: 125 and 127 n. 6, 2007: 42.

[39] Barker 1989: 125, 2007: 41–2. Barker (2007: 141) writes that 'the conception of pitch as inhabiting a dimension analogous to geometrical space was implicit, long before Aristoxenus, in the approaches taken by the *harmonikoi*, and was graphically represented in their diagrams, where pitches were set out as points marked on a line, and the intervals were represented by the gaps between them.'

musical interval (δίεσις) is an indivisible unit and principle of larger intervals, so an indemonstrable premise is an indivisible unit and principle of the theorems demonstrated from it (84b35–85a1).[40] Demonstrable theorems can thus be represented by the same one-dimensional diagrams that are used to represent musical intervals:



In this diagram, the intervals representing the indemonstrable premises are indivisible constituents of the interval representing the theorem demonstrated from them. Accordingly, Aristotle regards indemonstrable premises as elements (στοιχεῖα) of the theorems demonstrated from them:

> It is evident that when A holds of B, then if there is some middle term it is possible to prove that A holds of B, and the elements of this [conclusion] are these premises and they are as many as the middles;[41] for the immediate premises are

[40] See also *Metaph.* Δ 6 1016b17–24, I 1 1053a12–21, I 2 1053b34–1054a1; cf. Zabarella 1608: 938, Barker 1989: 70 n. 10 and 72 n. 16.

[41] καὶ στοιχεῖα τούτου ἔστι ταῦτα καὶ τοσαῦθ' ὅσα μέσα ἐστίν, 84b20–1. It is widely agreed that τούτου refers to the conclusion of the demonstration, AaB (Pacius 1597a: 477, Owen 1889: 297, Diels 1899: 29, Mure 1928: *ad loc.*, Tricot 1938: 121, Detel 1993: ii 405–6). I do not agree with those who take τούτου to refer to the demonstration of the conclusion (Tredennick 1960: 131, Mignucci 1975: 504). Also, some

elements, either all of them or the universal ones. But if there is no middle term, there is no longer a demonstration; but this is the path to the principles. (*APost.* 1.23 84b19–24)

In this passage, Aristotle states that each of the immediate premises AaC_1, C_iaC_{i+1}, and C_7aB is an element of the conclusion AaB. According to *Metaphysics* Δ 3, an element is a first, indivisible constituent of which something is composed (σύγκειται).[42] In the above diagram, the indemonstrable premises satisfy this description relative to the theorem demonstrated from them. Hence they can be viewed as elements of the theorem in much the same way that, say, the letters 'A' and 'B' are elements of the syllable 'BA'. Conversely, a demonstrable theorem is composed (σύγκειται) of the indemonstrable premises from which it is demonstrated. As such, the theorem is a composite (σύνθετον).[43] It is, as Zabarella puts it, 'composite because it consists of those propositions into which it can be

commentators replace ταῦτα at 84b21 by ταὐτά (Ross 1949: 585, Mignucci 1975: 503–4), or excise the phrase ταῦτα καὶ (Barnes 1994: 34). However, ταῦτα καὶ can be retained if it is taken to refer to the indemonstrable premises obtained by the insertion of middle terms between A and B (Mure 1928: *ad loc.*, Detel 1993: ii 406–7).

[42] *Metaph.* Δ 3 1014a26–30; see Crowley 2005: 370–3.

[43] Given that indemonstrable premises are elements of a theorem, it follows that the theorem is composed (σύγκειται, *Metaph.* Δ 3 1014a26) of these elements, and hence is a σύνθετον (*Metaph.* N 2 1088b14–16).

resolved', whereas an immediate proposition 'cannot be divided into other propositions'.[44]

Aristotle makes it clear that this account applies to demonstrations but not to deductions in general. An indemonstrable proposition is the conclusion of numerous sound deductions, but they fail to be demonstrations because the premises are not prior to and explanatory of the conclusion. In the passage just quoted, Aristotle excludes such deductions from consideration on the grounds that they are not part of 'the path to the principles' (ἡ ἐπὶ τὰς ἀρχὰς ὁδὸς).[45] In other words, they are not deductions in which the transition from the conclusion to the premises is a step towards the principles of a given science.

Finally, a comment is in order on Aristotle's remark that there are as many elements as middle terms (μέσα, 84b20–1).[46] As it stands, this remark is not entirely correct. The number of ultimate premises in a deduction is one more than the number of middle terms: one middle term gives rise to two premises, two middle terms to three

[44] Zabarella 1608: 938; similarly, Crubellier 2008: 126. Likewise, harmonicists distinguish between composite and incomposite musical intervals: an incomposite interval is one bounded by successive notes, and a composite interval is one that is 'composed of parts, into which it may also be divided' (Aristox. *Elem. Harm.* 75.11–16, see also 21.17–21, 76.1–9; cf. Cleonides 5.20–36, Arist. Quint. *De Mus.* 1.7 4–8).

[45] See Zabarella 1608: 934 and 940, Waitz 1846: 362.

[46] These μέσα are not intermediate intervals but middle terms; cf. στοιχεῖα τοσαῦτ' ἔστιν ὅσοι ὅροι, 84b26–7.

premises, and so on.[47] Perhaps for this reason, Aristotle adds the qualification that the elements in question are 'either all immediate premises or the universal ones'. He does not explain what the 'universal' premises are, but commentators take them to be those immediate premises that do not involve the minor term B.[48] So understood, Aristotle's remark that the elements are equal in number to the middle terms is correct.

Having discussed affirmative demonstrations in Barbara, Aristotle goes on to apply the terminology of elements to negative demonstrations:

> Similarly, too, if A does not hold of B, then if there is a middle or a prior term of which it does not hold, there is a demonstration; and if not, there is not, but it is a principle. And there are as many elements (στοιχεῖα) as terms; for the propositions consisting of these terms are principles of the demonstration. And just as there are some indemonstrable principles to the effect that this is this and this holds of this, so too there are some to the effect that this is not this and this does not hold of this. (*APost.* 1.23 84b24–30)

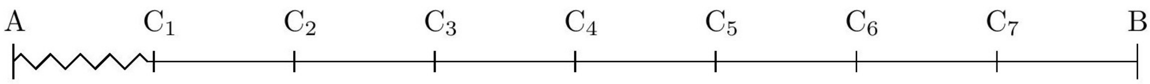
[47] This follows from the fact that, in every deduction, the number of ultimate premises is one less than the number of terms involved (*APr.* 1.25 42b1–16). Aristotle writes: 'the intervals (διαστήματα) are one fewer than the terms, and the premises are equal to the intervals' (42b9–10).

[48] Zabarella 1608: 933–4, Ross 1949: 585, Tredennick 1960: 132–3, Grosseteste 1981: 231–2. For an alternative proposal, see Detel 1993: ii 407.

In this passage, Aristotle considers demonstrations that establish a universal negative conclusion AeB ('A holds of no B'). In the first figure, such a demonstration takes the form of Celarent:

$$\frac{\mathrm{AeC} \qquad \mathrm{CaB}}{\mathrm{AeB}}$$

If the two premises are demonstrable, a full-blown demonstration tree can be constructed as follows (1.23 85a3–7):[49]



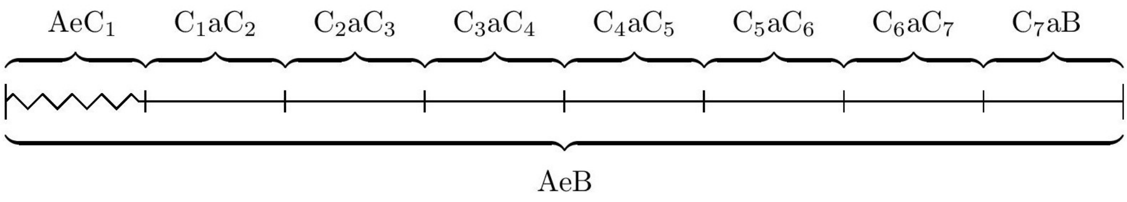
$$\cfrac{\cfrac{\cfrac{\mathrm{AeC}_1 \qquad \mathrm{C}_1\mathrm{aC}_2}{\mathrm{AeC}_2} \qquad \cfrac{\mathrm{C}_2\mathrm{aC}_3 \qquad \mathrm{C}_3\mathrm{aC}_4}{\mathrm{C}_2\mathrm{aC}_4}}{\mathrm{AeC}_4} \qquad \cfrac{\cfrac{\mathrm{C}_4\mathrm{aC}_5 \qquad \mathrm{C}_5\mathrm{aC}_6}{\mathrm{C}_4\mathrm{aC}_6} \qquad \cfrac{\mathrm{C}_6\mathrm{aC}_7 \qquad \mathrm{C}_7\mathrm{aB}}{\mathrm{C}_6\mathrm{aB}}}{\mathrm{C}_4\mathrm{aB}}}{\mathrm{AeB}}$$

Aristotle emphasizes that the premises at the top of this tree are indemonstrable principles, including the universal negative AeC_1.[50] Each of these indemonstrable premises is an element of the conclusion AeB. This conclusion can therefore be represented as a divisible interval composed of two kinds of indivisible interval representing indemonstrable a- and e-premises, respectively:[51]

[49] See Pacius 1597a: 479.

[50] Aristotle establishes the existence of indemonstrable e-premises in *Posterior Analytics* 1.15.

[51] Again, this fits the comparison with smallest musical intervals, since harmonicists employ two different kinds of smallest interval (διέσεις) as units to measure composite musical intervals (*Metaph.* I 1 1053a14–18; see Barker 1989: 73 n. 17).



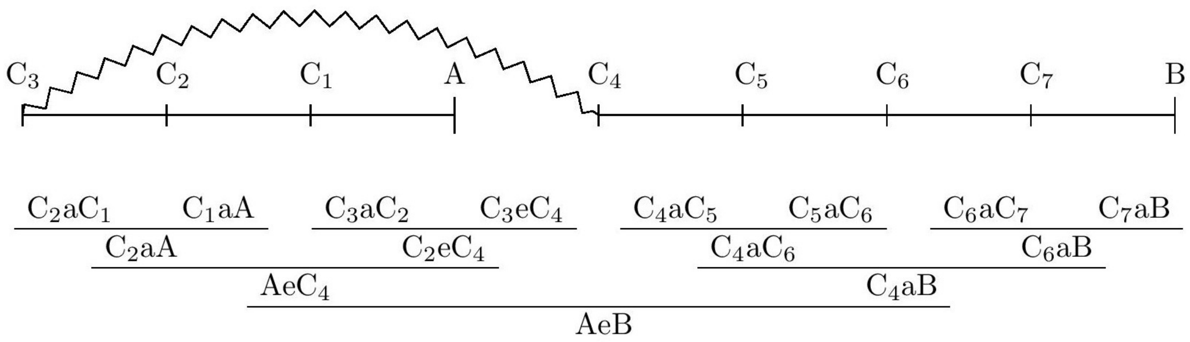
This type of diagram is applicable to all demonstrations that involve first-figure deductions in Barbara and Celarent. But is it also applicable to demonstrations that involve second- and third-figure deductions? Aristotle turns to this question in the final section of *Posterior Analytics* 1.23 (85a1–12). In doing so, he encounters a difficulty concerning the position of middle terms. Aristotle states that, if a demonstration establishes an a-conclusion AaB by repeated applications of Barbara, none of the middle terms 'falls outside' of the major term A (84b33–5 and 85a1–3). Presumably, none of them falls outside of the minor term B either, so that all middle terms fall within the interval between the major and minor terms.[52] The same is true for negative demonstrations that involve first-figure deductions in Barbara and Celarent (85a1–7).[53] However, this is not the case for second-figure deductions in Camestres:

$$\frac{\mathrm{CaA} \qquad \mathrm{CeB}}{\mathrm{AeB}}$$

[52] Ross 1949: 585, Grosseteste 1981: 233, Detel 1993: ii 408; *pace* Philop. *in APost.* 267.21–6.

[53] Pacius 1597a: 479, Zabarella 1608: 939–40, Ross 1949: 586, Mignucci 1975: 512, Detel 1993: ii 411.

Aristotle holds that, if a demonstration involves an application of Camestres, some middle terms may fall outside of the major term A (85a7–10).[54] For example, consider a demonstration in which the conclusion AeB is inferred by Celarent, whereas the intermediate e-conclusions AeC_4 and C_2eC_4 are inferred by Camestres:



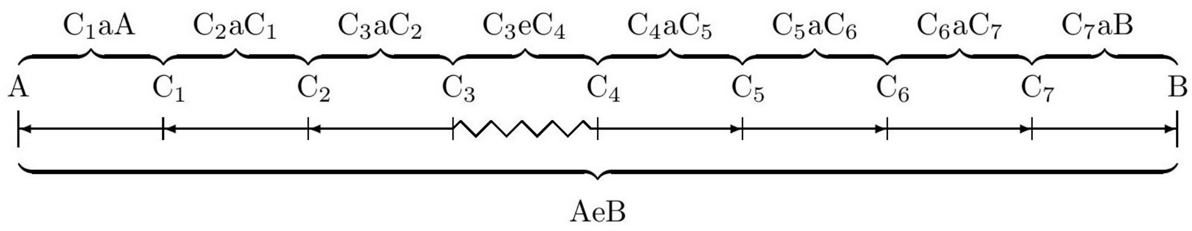
In this demonstration, both A and B are subject terms of a-premises. Consequently, if a-premises are represented by line segments in which the subject term always occurs on the right-hand side, then not all middle terms can be located between the extreme terms A and B. Thus, if A is located to the left of B, it follows that the middle terms C_{1-3} fall 'outside of' the major term A.

This poses a threat to Aristotle's diagrammatic representation of conclusions as one-dimensional intervals. For, if all a-premises are represented by line segments in which the subject term occurs on the right-hand side, the conclusion of the above

[54] Aristotle states that, if a demonstration involves a deduction in Camestres, no middle term falls outside of the minor term (85a7–10). While he does not explicitly state that some middle terms may fall outside of the major term, this is clearly implied by the context; see Philop. *in APost.* 270.20–2, Ross 1949: 586–7, Tredennick 1960: 134–5, Detel 1993: ii 411.

demonstration, AeB, cannot be represented as a one-dimensional interval between A and B such that all middle terms fall within this interval.[55] But if the conclusion cannot be represented as a composite consisting of indemonstrable premises, it is not clear on what grounds these premises can be regarded as elements of the conclusion.

Aristotle does not explain how to solve this problem in the *Posterior Analytics*. Still, we can provide a solution on his behalf, if a-premises are represented not simply by line segments but by directed line segments indicating the position of the subject term. The conclusion of the above demonstration can then be represented as follows:



In this diagram, a-premises are represented by arrows pointing toward the subject term. The first three are pointing left, the others are pointing right. Thus, the conclusion AeB can be represented as a one-dimensional interval composed of its ultimate premises, albeit one composed of directed line segments pointing in opposite directions. In this way, Aristotle is able to view these premises as elements of the conclusion, even though

[55] The same problem arises if all a-premises are represented by line segments in which the subject term occurs on the left-hand side. In this case, $C_{4–7}$ would fall outside of B. However, if a demonstration involves only applications of Camestres and Barbara but not of Celarent, then all middle terms can be viewed as falling within the interval between A and B; see n. 58.

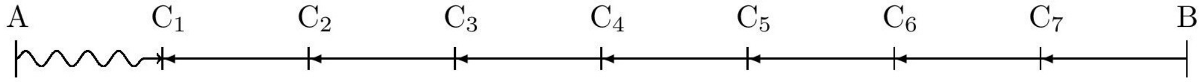
some middle terms fall 'outside of' the major term when a-premises are represented by undirected line segments.

Aristotle concludes chapter 1.23 by commenting on demonstrations that employ third-figure deductions in Bocardo (85a10–12).[56] These establish a particular negative conclusion AoB ('A does not hold of some B'):

$$\frac{\text{AoC} \qquad \text{BaC}}{\text{AoB}}$$

Repeated application of Bocardo yields complex demonstrations such as the following:[57]

[56] He describes these deductions by the phrase τρίτος τρόπος (85a10–11). This phrase is used to refer to third-figure deductions in Bocardo at *APost.* 1.21 82b21–8. Nevertheless, Ross and others take τρίτος τρόπος at 85a10–12 to refer to second-figure deductions in Cesare (Ross 1949: 587, Tredennick 1960: 135, Mignucci 1975: 512–14, Smith 1982a: 117, 1982b: 335, Barnes 1994: 173). They do so mainly because they think that Aristotle does not discuss any deductions with an o-conclusion in *Posterior Analytics* 1.23. Accordingly, they delete the reference to Baroco in 1.23 by excising ἢ μὴ παντί at 85a9 (Ross 1949: 587, Tredennick 1960: 134, Detel 1993: ii 411, Barnes 1994: 183). Barnes even excises 82b21–8 (see n. 57). If the manuscript reading is retained in these two passages, Ross's interpretation of 85a10–12 is not tenable. Traditionally, τρίτος τρόπος at 85a10–12 has been taken to refer to deductions in the third figure (Philop. *in APost.* 270.24–8, 232.1–25, Pacius 1597a: 479, Zabarella 1608: 940, Waitz 1846: 364, Owen 1889: 299, Mure 1928: *ad loc.*, Tricot 1938: 124). On the other hand, the three syllogistic figures are usually designated not by τρόπος but by σχῆμα (e.g., *APost.* 1.3 73a14–15, 1.13 78b24, 1.14 79a17–32, 1.15 79b15, 1.16 80a9, 80a27, 1.17 81a5). Thus, Crager (2015: 104–20) argues that τρίτος τρόπος at 82b22 designates not the third figure but a deductive pattern that is exemplified only by Bocardo. In any case, 85a10–12 deals only with negative deductions. In the third figure, these are Felapton, Ferison, and Bocardo. Thus, 85a10–12 deals either with these three moods or just with Bocardo.



$$\dfrac{\dfrac{\dfrac{\mathrm{AoC}_1 \quad \mathrm{C}_2\mathrm{aC}_1}{\mathrm{AoC}_2} \quad \dfrac{\mathrm{C}_4\mathrm{aC}_3 \quad \mathrm{C}_3\mathrm{aC}_2}{\mathrm{C}_4\mathrm{aC}_2}}{\mathrm{AoC}_4} \quad \dfrac{\dfrac{\mathrm{BaC}_7 \quad \mathrm{C}_7\mathrm{aC}_6}{\mathrm{BaC}_6} \quad \dfrac{\mathrm{C}_6\mathrm{aC}_5 \quad \mathrm{C}_5\mathrm{aC}_4}{\mathrm{C}_6\mathrm{aC}_4}}{\mathrm{BaC}_4}}{\mathrm{AoB}}$$

As before, the conclusion of this demonstration, AoB, can be represented as a one-dimensional interval bounded by A and B:[58]

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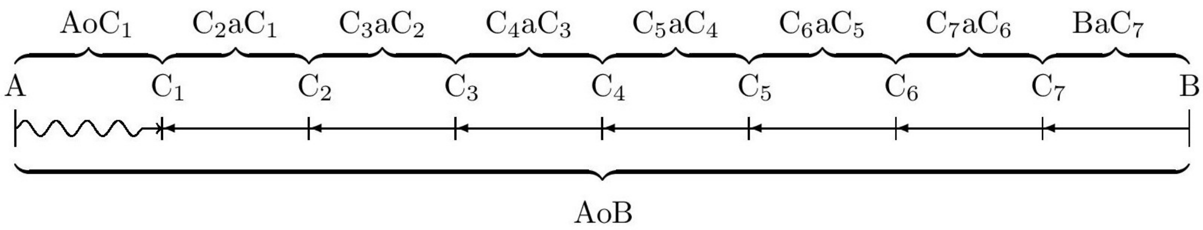
[57] Aristotle describes such repeated applications of Bocardo at *APost.* 1.21 82b21–8. Barnes (1994: 173) excises this passage, but his reasons for doing so are not compelling. His main argument is that τρίτος τρόπος at 82b22 cannot refer to Bocardo because this phrase is applied at 82b15–16 to deductions that infer an e-conclusion. However, the occurrence of τρίτος τρόπος at 82b15–16 can be understood as indicating a purely theoretical option of deriving negative conclusions without implying that an e-conclusion can actually be derived by through a deduction in the τρίτος τρόπος (Zabarella 1608: 904; cf. Philop. in *APost.* 232.3–6).

[58] Aristotle states that no middle term in this demonstration falls outside of the major or minor terms (85a10–12). This differs from his statement in *Prior Analytics* 1.6 that the middle term of third-figure deductions falls outside of both the major and the minor terms (28a14–15). Ross (1949: 586) suggests that in *Posterior Analytics* 1.23 a middle term C is said to fall outside of the major term A if CaA, and that it is said to fall outside of the minor term B if BaC (similarly, Mignucci 1975: 511–12, Detel 1993: ii 408–12). However, this conflicts with Aristotle's claim that the middle term of Bocardo does not fall outside of the minor term (85a10–12). The latter claim can be accommodated by modifying Ross's proposal as follows: C falls outside of the major term A if CaA and there is another middle term C_i such that C_iaB; and C falls outside of the minor term B if BaC and there is another middle term C_i such that AaC_i. On this account, no middle term falls outside of the minor term in the case of Bocardo (because there is no C_i such that AaC_i). By contrast, in the above demonstration employing Camestres and Celarent, three middle terms fall outside



This diagrammatic representation of conclusions is applicable not only to the demonstrations discussed by Aristotle in *Posterior Analytics* 1.23, but to all demonstrations that consist of deductions in the three syllogistic figures. This is because Aristotle's syllogistic theory satisfies a chain principle to the effect that the premises of any deduction form a chain of predications linking the terms of the conclusion. More precisely, let 'AB' denote a proposition of any of the four Aristotelian forms a, e, i, o, regardless of whether the subject term is A or B. Then the principle states that in any deduction inferring a conclusion AB the premises are of the form:[59]

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of the major term (whereas no middle term falls outside of the major term if the demonstration employs Camestres and Barbara but not Celarent).

[59] Smiley 1973: 139–45, 1994: 27, Thom 1981: 181–3. Aristotle can be seen to state this chain principle at *APr.* 1.23 40b30–41a20; cf. Smiley 1994: 29–34. The principle implies that, if two theorems are not both of the form AB, they cannot be demonstrated from the same set of indemonstrable premises. Thus, every class of theorems of the form AB has its own set of indemonstrable premises. Accordingly, Aristotle holds that the indemonstrable premises 'are not much fewer than the conclusions' (*APost.* 1.32 88b3–4). In this respect, his theory of demonstration differs from modern axiomatic systems, in which a large number of conclusions is derived from a relatively small number of ultimate premises.

$AC_1, C_1C_2, C_2C_3, C_3C_4, \ldots, C_{n-1}C_n, C_nB.$

Given this principle, and given that premises are represented by directed line segments pointing in either direction, Aristotle's diagrammatic representation of conclusions is applicable to any demonstration that proceeds in the three syllogistic figures. Thus, the conclusion of every demonstration can be represented as a one-dimensional interval bounded by the major and minor terms, and composed of indivisible directed line segments representing the indemonstrable premises from which the conclusion is demonstrated.[60] Conversely, indemonstrable premises are represented as indivisible constituents of the theorems demonstrated from them. This allows Aristotle to regard indemonstrable premises as elements (στοιχεῖα) of these theorems.

4. Elements are material causes

Given that indemonstrable premises are elements of the theorems demonstrated from them, does it follow that they are material causes of these theorems? I will argue that the answer is yes, on the grounds that every element of a thing is a material cause of that thing. To begin with, consider Aristotle's definition of element in *Metaphysics* Δ 3:

> An element (στοιχεῖον) is called that out of which as a primary constituent something is composed (ἐξ οὗ σύγκειται πρώτου ἐνυπάρχοντος), while being indivisible in form into another form. For example, the elements of a spoken

[60] Similarly, Smyth 1971: 485–6.

sound are the things of which the spoken sound is composed and into which it is ultimately divided, while they are no longer divided into other spoken sounds different in form from them. (*Metaph.* Δ 3 1014a26–30)

According to this definition, *x* is an element of *y* just in case *x* is a primary constituent of *y*, *y* is composed of *x*, and *x* is not divisible into items that differ from it in form. This is parallel to Aristotle's definition of material cause in *Metaphysics* Δ 2 as

> that from which as a constituent something comes to be (τὸ ἐξ οὗ γίγνεταί τι ἐνυπάρχοντος). (*Metaph.* Δ 2 1013a24–5 = *Phys.* 2.3 194b24)

Thus, *x* is a material cause of *y* just in case *x* is a constituent of *y* and *y* comes to be from *x*. Moreover, Aristotle holds that everything composed of elements comes to be from these elements.[61] Consequently, whatever is an element of a thing is a material cause of that thing.[62]

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[61] In *Metaphysics* N 2 (1088b14–17), Aristotle argues that everything composed of elements (ἐκ στοιχείων συγκεῖσθαι) comes to be from these elements, since 'necessarily, a thing comes to be from that of which it consists, whether it is eternal or whether it came to be' (ἀνάγκη, ἐξ οὗ ἐστιν, εἰ καὶ ἀεὶ ἔστι, κἄν, εἰ ἐγένετο, ἐκ τούτου γίγνεσθαι); see Bonitz 1849: 573–4. Cf. *Metaph.* N 5 1092a29–32 (on which see Annas 1976: 217).

[62] Aquinas *in Metaph.* 795.

This view is shared by Aristotle's pupil Eudemus, who holds that 'while 'cause' is said in four ways, 'element' is said in the sense of matter'.[63] Likewise, Alexander takes 'an element to be that which is a constituent as matter'.[64] By contrast, Simplicius disagrees with them, pointing to passages in *Metaphysics* Λ 4 and *Physics* 1.6 in which Aristotle identifies three principles that he calls 'elements': form (εἶδος), privation, and matter.[65] The fact that form counts as an element, Simplicius argues, shows that not every element is a material cause; for presumably form is not a material but a formal cause.[66] Simplicius takes Aristotle to hold that the form of a hylomorphic compound is an element of this hylomorphic compound.[67] However, Aristotle does not explicitly express this view.[68] On the contrary, he explicitly denies it in *Metaphysics* Z 17. In this chapter, Aristotle emphasizes that, while the letters 'B' and 'A' are elements of the syllable 'BA', the form of the syllable is not an element of the syllable.[69] Thus, his pronouncements in Λ 4 and *Physics* 1.6 should not be taken to commit him to the view that form is an element *of* the hylomorphic compound of which it is the form. Instead, he might call form an 'element'

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[63] Simpl. *in Phys.* 10.13–14 = Eudemus fr. 32 Wehrli.

[64] Simpl. *in Phys.* 10.12, see also 13.21–2.

[65] *Metaph.* Λ 4 1070b10–30, *Phys.* 1.6 189b16–29; cf. Bonitz 1849: 226, Crubellier 2000: 144, Code 2015: 19–21.

[66] See Simpl. *in Phys.* 11.21–3, 13.28–33.

[67] Simpl. *in Phys.* 11.21–3, 13.31–3; similarly, Philop. *A Treatise Concerning the Whole and the Parts* 92.2 (transl. King 2015: 205).

[68] Although he seems to come close to it at *Metaph.* Λ 5 1071a13–15 (see Ross 1924: ii 364).

[69] *Metaph.* Z17 1041b11–33; cf. n. 110 below.

for some other reason, perhaps under the influence of similar uses of 'element' in the Academy.[70]

Given Aristotle's argument in *Metaphysics* Z 17, there is little doubt that, in his considered view, the form of a thing is not an element of this thing, and hence that every element of a thing is a material cause of it. This is confirmed by the final sentence of Z 17:

> An element is that into which a thing is divided, a constituent present in it as matter (εἰς ὃ διαιρεῖται ἐνυπάρχον ὡς ὕλην); for example, A and B are elements of the syllable. (*Metaph*. Z 17 1041b31–3).

Similarly, Aristotle writes:[71]

> The elements are matter of a substance (τὰ δὲ στοιχεῖα ὕλη τῆς οὐσίας). (*Metaph*. N 2 1088b27)

Given that indemonstrable premises of demonstrations are elements of the theorems demonstrated from them, it follows that these premises are material causes of

[70] Crubellier 2000: 142–4. We know that Plato took both matter and form to be elements and elemental principles (στοιχειώδεις ἀρχὰς); apud Simpl. *in Phys*. 7.24–7, 179.12–14, 223.10–16, 245.9. Moreover, Aristotle criticizes the Platonists for 'making every principle an element' (*Metaph*. N 4 1092a5–7); cf. Menn 2001: 102–6 and 127–34, Crowley 2005: 373.

[71] See also *Metaph*. M 8 1084b9–10; cf. Ps.-Alex. Aphr. *in Metaph*. 773.16–21.

the theorems. This, I submit, is what Aristotle had in mind when he claims in *Physics* 2.3 and *Metaphysics* Δ 2 that hypotheses are material causes of the conclusion.

By contrast, it is not the case that the premises of every deduction are elements of the conclusion. This is because Aristotle requires that elements be in some way prior to that of which they are elements:

> The element is prior (πρότερον) to the things of which it is an element.[72] (*Metaph.* Λ 4 1070b2–3)

Demonstrations display a priority ordering among propositions in that every premise is prior to the conclusion.[73] But the premises of a deduction need not be prior to the conclusion. Aristotle holds that for every deduction inferring AaB from AaC and CaB, there is another deduction using the conclusion AaB to infer one of the premises, as in:[74]

AaB, BaC, therefore AaC

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[72] Similarly, *Metaph.* M 10 1087a3–4. An element is prior to that of which it is an element ὡς ὕλη, whereas a compound is prior to its elements ὡς κατὰ τὸ εἶδος (*Metaph.* M 8 1084b9–13). For our purposes, it is not necessary to enter into a discussion of these senses of priority. What is important is that, in any given domain, there is a uniform sense of priority such that the elements of the domain are prior in that sense to the things of which they are elements.

[73] *APost.* 1.2 71b19–72a5, 1.25 86a39–b5. Aristotle assimilates this sense of priority to the one in which letters are prior to syllables, classifying both as priority in order (τῇ τάξει, *Cat.* 12 14a36–b3).

[74] *APr.* 2.5 57b18–28; cf. *APost.* 1.13 78a28–b4.

If every premise of a deduction were prior to the conclusion, then the proposition AaC would be prior to AaB and vice versa. But since priority is an asymmetric relation, this is impossible (*APost*. 1.3 72b25–32). Thus, unlike demonstrations, deductions cannot display a uniform priority ordering among propositions. Consequently, the premises of every deduction cannot be regarded as elements of its conclusion.

Likewise, it would be difficult for Aristotle to accept that every premise of a deduction is a material cause of the conclusion. For, in the example just discussed, this would imply that AaC is a material cause of AaB and vice versa, contradicting Aristotle's contention that no two items can be material causes of one another (*Phys.* 2.3 195a8–11). This is a problem for the traditional reading of Aristotle's claim in *Physics* 2.3 and *Metaphysics* Δ 2, on which the claim applies to all premises of deductions.[75] One might

[75] In *Posterior Analytics* 2.11, Aristotle refers to the material cause by the phrases 'given what things is it necessary for this to be' (τίνων ὄντων ἀνάγκη τοῦτ' εἶναι, 94a21–2) and 'given some thing, it is necessary for this to be' (οὗ ὄντος τοδὶ ἀνάγκη εἶναι, 94a24); see Themistius *in APost.* 52.1–11, Ps.-Philop. *in APost.* 375.32–376.2, 376.19–21, Eustratius *in APost.* 139.11–32, Zabarella 1608: 1147, Waitz 1846: 402, Owen 1889: 333, Mure 1928: ad 94a21–2, Balme 1972: 83–4, Barnes 1994: 226–7, Leunissen 2010: 182–6, *pace* Ross 1949: 638–42 and Dancy 1978: 374–7. These genitive-absolute phrases are often taken to invoke Aristotle's claim in *Physics* 2.3 and *Metaphysics* Δ 2 that hypotheses are material causes of the conclusion; Waitz 1846: 402, Mure 1928: ad 94a21–2, Balme 1972: 83, Sorabji 1980: 51 n. 24, Hankinson 2009: 216. Now, Aristotle often uses such genitive-absolute phrases to express *necessitas consequentiae*, i.e., the type of necessity with which the conclusion of any deduction follows from the premises (e.g., *APr.* 1.10 30b31–40, 1.15 34a5–24, 2.2 53b11–20, 2.4 57a40–b3). Thus, their use at 94a21–4 might be taken to imply that the premises of all deductions are material causes of the conclusion (see Zabarella 1608: 1149, Ross 1924: i 292–

argue that the claim should be understood as relativized to deductions, with premises being material causes of a conclusion not *simpliciter* but only relative to the deduction in which the conclusion is inferred from them. However, there is no evidence that Aristotle wished to relativize the claim in this way. Nor is there evidence that he regarded premises as material causes of the conclusion not without qualification but only *qua* conclusion of a certain deduction. By restricting the claim to indemonstrable premises of demonstrations, Aristotle is able to regard these premises as material causes of the conclusion *simpliciter*, without any qualification.

In *Posterior Analytics* 1.29, Aristotle discusses cases in which a single proposition is demonstrated by two demonstrations using different middle terms. He expresses scepticism about such cases later on (2.17 99b4–7), and it is not clear whether he ultimately accepts them. But if he does, then one proposition can be the conclusion of two or more demonstrations from different sets of indemonstrable premises. On the account just given, each of these indemonstrable premises is a material cause of the proposition *simpliciter*, not relative to a demonstration. Thus, every indemonstrable

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3). However, this is not correct. Aristotle goes on to illustrate the material cause by the questions, 'Why is the angle in the semicircle right? Given what thing, is it right?' (διὰ τί ὀρθὴ ἡ ἐν ἡμικυκλίῳ; τίνος ὄντος ὀρθή; 94a28). The genitive absolute in the second question does not express mere *necessitas consequentiae*, since premises such as 'Every angle in a semircle is a stone' and 'Every stone is right' clearly do not count as an adequate answer to the question. Instead, Aristotle's focus in *Posterior Analytics* 2.11 is on deductions that impart scientific knowledge, i.e., on demonstrations (see 94a20–4). Thus, the genitive-absolute phrases in *Posterior Analytics* 2.11 do not express mere *necessitas consequentiae* (although it is beyond the scope of this paper to examine their precise meaning).

premise that occurs in *some* demonstration of a given proposition, is a material cause of that proposition.

This does not necessarily mean that the material causes of a demonstrable proposition include only indemonstrable premises. It is open to Aristotle to take demonstrable propositions that serve as intermediate conclusions in the demonstration of a theorem to be material causes of this theorem. For example, if a demonstrable proposition AaC_4 is used as an intermediate conclusion in demonstrating the theorem AaB, then it may be regarded as a material cause of the theorem, just as the syllable 'BA', say, can be regarded as a material cause of the word 'ΒΑΘΟΣ'. As we will see in the next section, Aristotle seems to accept this view when he takes the elementary theorems of a given science to be material causes of the more complex theorems demonstrated by means of them.

5. Geometrical analysis: decomposing theorems

In the preceding I have emphasized Aristotle's claim in *Posterior Analytics* 1.23 that indemonstrable premises are elements of the theorems demonstrated from them. It must be admitted that this claim does not appear elsewhere in the *Analytics*. In fact, apart from chapter 1.23 Aristotle does not use the term 'element' in the *Analytics*. Moreover, the claim in 1.23 depends on the technical framework of Aristotle's syllogistic theory, and presupposes familiarity with the argument of chapters 1.19–22. It is unlikely that Aristotle took all of this to be common knowledge among the intended audience of works such as the *Physics* and *Metaphysics*. Thus, it is doubtful that he tacitly relies on the discussion in 1.23 when giving the list of material causes in *Physics* 2.3 and *Metaphysics* Δ 2 – especially since he seems to take this list to be intelligible to a wide audience without further explanation.

In what follows, I argue that, even independently of the discussion in *Posterior Analytics* 1.23, Aristotle was able to presuppose a well-established use of 'element' in which it refers to premises of demonstrations. As we will see, this use of 'element' is common in Aristotle's works other than the *Analytics* and was also familiar to mathematicians of his time. Consider, for example, the following passage from the *Categories*:

> A thing is called prior with respect to some order, as in the sciences and speeches. For in the demonstrative sciences there is a prior and posterior in order, for the elements are prior in order to the geometrical theorems (τῶν διαγραμμάτων), and in grammar the letters are prior to the syllables. (*Cat.* 12 14a36–b2)

It is widely thought that the διαγράμματα referred to in this passage are not figures or diagrams but geometrical theorems.[76] As Philoponus points out, ancient geometers called

[76] Philop. *in Cat.* 192.20–193.5, Heiberg 1904: 6, Ross 1924: i 234, Burkert 1959: 190, Ackrill 1963: 111, Knorr 1975: 72, Acerbi 2008: 540 n. 107. Similarly, Heath (1949: 216) takes these διαγράμματα to be 'geometrical propositions including the proofs of the same'.

the theorems of geometry διαγράμματα.[77] Accordingly, the elements mentioned in the passage are taken to be premises from which these theorems are demonstrated.[78] More specifically, they are taken to be either indemonstrable premises or elementary theorems of geometry. This agrees with a similar use of 'element' in *Metaphysics* B 3:

> The elements and principles of spoken sound are thought to be the primary constituents of which spoken sounds are composed . . .; and we call elements of geometrical theorems (τῶν διαγραμμάτων) those theorems the demonstrations of which are constituents of the demonstrations of the others, either of all or of most.[79] (*Metaph.* B 3 998a23–7)

Again, the διαγράμματα referred to in this passage are geometrical theorems.[80] The elements in question are the sort of thing of which there is a demonstration, viz., propositions. Specifically, they are elementary theorems of geometry whose

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77 Philop. *in Cat.* 193.2–5; see Knorr 1975: 72–3.

78 Waitz 1844: 317, Diels 1899: 26–7, Burkert 1959: 190–1, Ackrill 1963: 111.

79 καὶ τῶν διαγραμμάτων ταῦτα στοιχεῖα λέγομεν ὧν αἱ ἀποδείξεις ἐνυπάρχουσιν ἐν ταῖς τῶν ἄλλων ἀποδείξεσιν ἢ πάντων ἢ τῶν πλείστων. The phrase τῶν διαγραμμάτων might be taken to depend on ταῦτα (Burkert 1959: 190), but the parallel passages at 998a23, 998a28, and 1014a35–6 suggest that it depends on στοιχεῖα (Bonitz 1890: 44). Given that the phrase refers to geometrical theorems, it can also be construed as having both roles.

80 Asclep. *in Metaph.* 174.9–14, Heiberg 1904: 6, Ross 1924: i 234, Einarson 1936: 41, Heath 1949: 205, Burkert 1959: 190, Acerbi 2008: 540 n. 107; similarly, Bonitz 1849: 150–1, Berti 2009: 110.

demonstration is used in the demonstrations of all or most other geometrical theorems.[81] According to Proclus, this use of 'element' was common among ancient mathematicians.[82] It is also why Euclid's *Elements* is so called – because it establishes elementary theorems that constitute the basis for the more complex theorems of geometry.[83]

Although elementary theorems are demonstrable, they may be assumed without demonstration in the proof of other theorems if they are known to the audience.[84] In such contexts, they play a role analogous to that of indemonstrable premises. Since they are demonstrable, they are not hypotheses without qualification. Instead, Aristotle regards them as 'hypotheses in relation to a learner', by which he means demonstrable propositions assumed without demonstration by an instructor on the grounds that they are accepted as true by the learner(s) in the audience.[85] For example, if a demonstrable proposition AaC_4 is accepted as true by the learner(s), perhaps because it was demonstrated in a previous lecture, then the instructor may assume it without demonstration as a premise in establishing AaB. In this case, AaC_4 is a hypothesis in

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[81] Aquinas *in Metaph.* 424, Ross 1924: i 233–4, Burkert 1959: 190.

[82] Proclus *in Eucl. Elem.* 71.27–72.13; cf. Burkert 1959: 189. For example, this use of 'element' is found in Archimedes (*On Conoids and Spheroids* 164.13, *Quadrature of the Parabola* 165.26–7, 167.10; cf. Mugler 1958: 380). Similarly, Aristox. *Elem. Harm.* 37.4 and 54.13; cf. Westphal 1883: 183–7, Barker 1989: 123.

[83] Proclus *in Eucl. Elem.* 73.5–14 and 71.24–72.19; see also Alex. Aphr. *in Metaph.* 202.12–17.

[84] Heath 1949: 206.

[85] *APost.* 1.10 76b27–30; see Cornford 1932: 39–40, Lee 1935: 116, von Fritz 1955: 37, McKirahan 1992: 44–6, *pace* Zabarella 1608: 798–9.

relation to the learner(s). When Aristotle states that hypotheses are material causes of the conclusion, he may primarily have in mind indemonstrable hypotheses without qualification; but he may also have in mind elementary theorems that play the role of relative hypotheses in a given teaching context. This, at any rate, is suggested by the fact that he refers to elementary theorems as 'elements' in the passage just quoted.[86] If this is correct, the material causes of a theorem include not only indemonstrable premises but also the intermediate conclusions by means of which it is demonstrated.

In *Metaphysics* Δ 3, Aristotle distinguishes several uses of the term 'element', including one in which it designates 'the elements of geometrical theorems' (τὰ τῶν διαγραμμάτων στοιχεῖα, 1014a35–6). As before, these are premises, demonstrable or not, which are regarded as elements of the theorems demonstrated from them.[87] Aristotle makes it clear that this is not a metaphorical use of 'element', but an ordinary use that conforms to his official definition of an element as 'that out of which as a primary constituent something is composed, while being indivisible in form into another form'

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[86] In *Metaphysics* B 3, Aristotle lists four examples of elements: (i) letters of spoken sounds, (ii) elementary geometrical theorems of more complex theorems, (iii) earth, air, fire, and water of bodies, (iv) component parts of composites (998a23–b3; see Sedley 2004: 155–6). These correspond exactly to four of the five examples of material causes listed in *Physics* 2.3 and *Metaphysics* Δ 2: (i) letters of syllables, (ii) matter of artefacts, (iii) fire and the like of bodies, (iv) parts of the whole, (v) hypotheses of the conclusion. This suggests that Aristotle took all four examples listed in B 3 to be examples of material causes.

[87] Ross 1924: i 233–4 and 295, Crowley 2005: 371–5. Aristotle goes on to describe a related but distinct use of 'element', in which simple demonstrations are called elements of more complex demonstrations of which they are constituents (1014a36–b3).

(1014a26–7).[88] Notably, he does not introduce it as a new use of 'element' but seems to assume that it is familiar to his audience. He is able to do so because this use was known to mathematicians such as Menaechmus, a pupil of Eudoxus associated with Plato and the Academy. Proclus reports that Menaechmus distinguished two uses of 'element' in mathematics, the second of which is characterized as follows:

> 'Element' is said in two ways, as Menaechmus tells us. . . . In another way, an element is called a simpler constituent into which a composite (σύνθετον) can be divided. In this sense, not everything can be called an element of anything [that follows from it[89]], but only the more principle-like members of an argument explaining a conclusion, as postulates are elements of theorems.[90] (Proclus *in Eucl. Elem.* 72.23–73.9)

[88] He lists it among the ordinary uses of 'element' at 1014a27–b3, before turning to metaphorical (μεταφέροντες) uses at 1014b3–14; see Crowley 2005: 371–5 (similarly, Aquinas *in Metaph.* 795–804).

[89] In the first use of 'element' identified by Menaechmus, a proposition counts as an element of another if the former can be used as a premise in a sound deduction deriving the latter (72.24–6; see Barnes 1976: 286–9). In this sense, two propositions can be elements of one another (72.26–73.5). Aristotle rejects such a notion of element (see Section 4 above; cf. Brown 1976: 262–9).

[90] ἀλλὰ τὰ ἀρχοειδέστερα τῶν ἐν ἀποτελέσματος λόγῳ τεταγμένων, ὥσπερ τὰ αἰτήματα στοιχεῖα τῶν θεωρημάτων. Proclus uses ἀποτέλεσμα to designate the effect of a cause (αἰτία); see, e.g., *Elem. Theol.* 75.1–12, *Theol. Plat.* ii 37.23–4, *in Eucl. Elem.* 61.8–10 (cf. Duvick 2007: 127 n. 120). Accordingly, an ἀποτελέσματος λόγος is a mathematical proof in which the premises are causal or explanatory of the conclusion.

According to this passage, a proposition is an element of a theorem if it is more principle-like (ἀρχοειδέστερα) than the theorem and if it appears in an explanatory proof of the theorem. While the 'postulates' mentioned at the end of the passage are indemonstrable principles of geometry, the preceding characterization of elements applies to elementary theorems and indemonstrable premises alike: both are elements of the theorems demonstrated from them.[91] As such, they are simpler constituents into which the theorem – a composite (σύνθετον) – can be divided. This corresponds exactly to Aristotle's notion of an element of theorems that we saw above. If Proclus' report is correct, this notion was familiar to Menaechmus and other mathematicians associated with the Academy in Aristotle's time.[92]

Both Menaechmus and Aristotle regard a theorem as a composite (σύνθετον) consisting of the premises from which it can be demonstrated. This is in accordance with the methods of analysis and synthesis in ancient geometry. For example, Alexander describes these methods as follows:[93]

> The reduction of any composite (συνθέτου) to the things from which it is composed is called analysis. Analysis is the converse of synthesis. Synthesis is the

[91] The comparative form ἀρχοειδέστερα suggests that the elements in question need not be principles (ἀρχαί).

[92] Burkert 1959: 191–2; cf. Crowley 2005: 375–6.

[93] Similarly, Pappus *Synag.* VII 634.3–636.14, Philop. *in APost.* 162.16–28.

> path from the principles to those things that derive from the principles, and analysis is the return path from the end up to the principles. Geometers are said to analyze when they begin from the conclusion and proceed in order through the things used for the demonstration of the conclusion and thereby ascend to the principles and the problem.[94] But you also use analysis if you reduce composite bodies to simple bodies. . . . Again, if you divide speech into the parts of speech, or the parts of speech into syllables, or these into letters, you are analyzing.
> (Alexander *in APr.* 7.12–22)

Geometrical synthesis is a path from indemonstrable principles to the theorems demonstrated from them, while geometrical analysis is the converse path from the latter

[94] οἵ τε γὰρ γεωμέτραι ἀναλύειν λέγονται, ὅταν ἀπὸ τοῦ συμπεράσματος ἀρξάμενοι κατὰ τὴν τάξιν τῶν εἰς τὴν τοῦ συμπεράσματος δεῖξιν ληφθέντων ἐπὶ τὰς ἀρχὰς καὶ τὸ πρόβλημα ἀνίωσιν. A πρόβλημα is a task to demonstrate a theorem or to perform a certain geometrical construction. As such, it is typically the object of analysis but not the endpoint of analysis. Thus, Barnes et al. (1991: 49–50) translate the clause as follows: '. . . when they begin from the conclusion and proceed in order through the assumptions made for the proof of the conclusion until they bring the problem back to its principles'. However, since this translation ignores the καὶ before τὸ πρόβλημα, I prefer the translation given above. On this reading, analysis need not reduce a geometrical problem or theorem exclusively to indemonstrable principles, but also to more elementary problems or theorems (for this kind of reductive analysis, see Heath 1921: 291, Menn 2002: 203 and 211–14). Accordingly, Hero and Pappus describe the endpoint of analysis as 'something whose proof has already preceded' (Hero, transl. Knorr 1986: 376 n. 83), and 'something that is established by synthesis' (Pappus *Synag.* VII 634.12–13; see Cornford 1932: 46, Hintikka & Remes 1974: 75–7, Knorr 1986: 354–5).

to the former (or to more elementary theorems). Alexander likens geometrical analysis to the division of syllables into letters and to the reduction of composite bodies to simple bodies. They are all instances of analysis in general, which consists in reducing a composite to the things from which it is composed. Thus, again, geometrical theorems are viewed as composites: just like composite bodies such as flesh are composed of simple bodies such as fire and earth, and just like syllables are composed of letters, so theorems are composed of the principles from which they are demonstrated.[95]

There is strong evidence that Aristotle was aware of the methods of geometrical analysis and synthesis described by Alexander. For example, he refers to them in the *Sophistici elenchi*:

> The same thing happens sometimes as with geometrical theorems, for there we sometimes analyze but are unable to compose it again.[96] (*Soph. el.* 16 175a26–8)

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[95] Beaney (2002: 55, 2007: 197–8) distinguishes geometrical analysis, which 'involves working back to the principles, premises, causes, etc. by means of which something can be derived or explained', from decompositional analysis, which 'involves identifying the elements and structure of something'. While Beaney takes them to be two different kinds of analysis, Alexander and Aristotle view geometrical analysis as a special case of decompositional analysis. Similarly, Ammonius argues that any object of analysis must be composite (σύνθετον), since analysis is the resolution of a composite into a plurality of simples (*in Porph. Isag.* 37.7–16).

[96] συμβαίνει δέ ποτε καθάπερ ἐν τοῖς διαγράμμασιν· καὶ γὰρ ἐκεῖ ἀναλύσαντες ἐνίοτε συνθεῖναι πάλιν ἀδυνατοῦμεν. It is agreed that this is a reference to geometrical analysis and synthesis (Pseudo-Michael *in*

In this passage, the verb 'compose' describes the demonstration of a theorem from suitable premises, and 'analyze' the converse transition from the theorem back to the premises.[97] The same use of 'analyze' occurs in the *Nicomachean Ethics* and *Posterior Analytics*:

> For the person who deliberates seems to inquire and analyze in the way described as though he were analyzing a geometrical theorem.[98] (*NE* 3.3 1112b20–1)

> If it were impossible to prove a truth from falsehood, it would be easy to analyze, for then the propositions would convert by necessity.[99] (*APost.* 1.12 78a6–8)

Soph. el. 122.19–23, Pacius 1597b: 510, Hintikka & Remes 1974: 87, Menn 2002: 207, Fait 2007: 176–7). As before, I take the διαγράμματα to be geometrical theorems (Knorr 1975: 72, Acerbi 2008: 540 n. 107).

[97] Similarly, Aristotle states that, when a demonstrator 'thinks the two premises, he thinks and composes the conclusion (τὸ συμπέρασμα ἐνόησε καὶ συνέθηκεν)', *MA* 7 701a10–11.

[98] ὁ γὰρ βουλευόμενος ἔοικε ζητεῖν καὶ ἀναλύειν τὸν εἰρημένον τρόπον ὥσπερ διάγραμμα. This is a reference to the method of geometrical analysis (Cornford 1932: 44, Einarson 1936: 36, Ross 1949: 549, Hintikka & Remes 1974: 32 and 85–7, Menn 2002: 208, Crubellier 2014: 23). Again, I take the διάγραμμα to be a geometrical theorem (Acerbi 2008: 540 n. 107).

[99] For Aristotle's discussion of geometrical analysis in this passage, see Philop. *in APost.* 162.14–164.4, Ross 1949: 548–50, Knorr 1986: 75–6, Menn 2002: 205–7.

Furthermore, Aristotle seems to allude to analysis in *Posterior Analytics* 1.23 when he speaks of 'the path to the principles' (ἡ ἐπὶ τὰς ἀρχὰς ὁδὸς, 84b23–4). This phrase corresponds to Alexander's characterization of geometrical analysis as the 'path from the end up to the principles' (ἐπάνοδος ἀπὸ τοῦ τέλους ἐπὶ τὰς ἀρχάς, *in APr.* 7.15). It also bears a close resemblance to the canonical descriptions of geometrical analysis given by Albinus and Pappus.[100] Thus, the bottom-up construction of complex demonstrations discussed by Aristotle in *Posterior Analytics* 1.23, is an instance of analysis as described by Alexander.[101]

Aristotle reports that Plato used to investigate whether, in a given argument, 'the path is from the principles or to the principles'.[102] This might be taken to indicate that Plato was concerned to distinguish the methods of synthesis and analysis in geometry and other disciplines. In fact, some sources credit Plato with inventing the method of

[100] Albinus: ἄνοδος ἐπὶ τὰς ἀναποδείκτους καὶ ἀμέσους προτάσεις (*Didasc.* 5.4 Louis). Pappus: ὁδὸς ἀπὸ τοῦ ζητουμένου ὡς ὁμολογουμένου διὰ τῶν ἑξῆς ἀκολούθων ἐπί τι ὁμολογούμενον συνθέσει (*Synag.* VII 634.11–13). Pseudo-Philoponus: ἡ ἀπὸ γνωστοῦ τινος εἰς τὰς ἀρχὰς καὶ τὰ αἴτια ἐπάνοδος (*in APost.* 335.28–9).

[101] Solmsen 1929: 121–3; cf. Zabarella 1608: 934. Accordingly, Crubellier (2008: 126) suggests that Aristotle's claim in *Physics* 2.3 and *Metaphysics* Δ 2 appeals to a conception of analysis on which 'the analyst divides the conclusion into two distinct propositions, which will be the premises' (similarly, Charlton 1970: 100; although I do not agree with Crubellier and Charlton that Aristotle's claim applies to the premises of all deductions).

[102] πότερον ἀπὸ τῶν ἀρχῶν ἢ ἐπὶ τὰς ἀρχάς ἐστιν ἡ ὁδός, *NE* 1.4 1095a32–3.

geometrical analysis and teaching it to Leodamas.[103] Whether or not this is correct, it is likely that the method was known to Plato and members of the Academy.[104] Thus, Aristotle was able to assume that his audience is familiar with the notion that theorems are analyzed and decomposed into the indemonstrable principles from which they are demonstrated, and that they are composites consisting of these principles as elements. Call this the compositional conception of theorems.

In *Posterior Analytics* 1.23, Aristotle elaborates the compositional conception by a diagrammatic model in which theorems are represented as one-dimensional intervals composed of indivisible directed line segments.[105] Since this model depends on the details of Aristotle's syllogistic theory, it may not have been generally known among philosophers and mathematicians of Aristotle's time. Instead, they might have endorsed the compositional conception of theorems on other grounds. For example, they might have been motivated by considerations such as those put forward by Socrates in the dream theory of the *Theaetetus*. This theory is concerned with the proposal that knowledge (ἐπιστήμη) is true judgment with an account (201c8–d2). The proposal

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[103] Diogenes Laertius 3.24, Proclus *in Eucl. Elem.* 211.18–23.

[104] See Cornford 1932: 43–8, Menn 2002: 205–15.

[105] More generally, in *Posterior Analytics* 1.23, Aristotle seeks to relate the results of chapters 1.19–22 to themes that were prominent in the Academy. This can be seen from his use in 1.23 of the Platonic phrase 'the path to the principles' and of the term 'element', which does not occur elsewhere in the *Analytics* but played a central role in discussions within the Academy concerning the principles of mathematics and other disciplines (see Diels 1899: 17–23, Burkert 1959: 191–7, Krämer 1973: 144–55, Menn 2001: 102–6, Berti 2009: 105–16).

implies that things that do not have an account are unknowable, while those that have an account are knowable (201d2–3). Socrates characterizes the former class of things as elements, and the latter as complexes composed of these elements:

> The primary elements (στοιχεῖα), as it were, of which we and everything else are composed, have no account. . . . But with the things composed of these, it is another matter. Here, just in the same way as the elements themselves are woven together, so their names, woven together, become an account of something; for an account is essentially a complex (συμπλοκὴν) of names. Thus, the elements have no account and are unknowable, but they are perceivable, whereas the complexes (συλλαβὰς) are knowable and expressible in an account and judgeable in a true judgement. (*Theaetetus* 201e1–202b7)

According to this theory, all objects of knowledge are complex in that they are composed of elements.[106] For an account of a thing is essentially a complex of names referring to its elements, so that anything that is not composed of elements fails to have an account and is therefore unknowable. In particular, since elements are not themselves composed of elements (205c4–7), they are unknowable (202b6, 202e1).

The dream theory is largely abstract in that it does not specify what kind of thing the elements and complexes are.[107] As such, the theory is open to various interpretations.

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[106] See Burnyeat 1990: 134–5.

[107] Morrow 1970: 328, McDowell 1973: 234, Burnyeat 1990: 135 and 145, Mann 2011: 43.

Socrates illustrates it by the model of letters and syllables (202e–203d), but the theory is clearly intended to be applicable to other domains as well. For example, Morrow suggests that it is applicable to the domain of mathematical propositions when the elements are taken to be indemonstrable premises of mathematics.[108] On this interpretation, the complexes composed of the elements are the theorems demonstrated from them. This is the compositional conception of theorems. Whether or not Plato had in mind this interpretation of the dream theory, it is perfectly possible that some of his readers in the Academy interpreted it in this way.[109] At least, we know that they referred to premises of mathematical demonstrations as 'elements'.

Thus, the dream theory of the *Theaetetus* provides further support for the compositional conception of theorems. It is the presence of this conception in the Academy, I submit, that allows Aristotle to state without further explanation that indemonstrable premises are constituents and material causes of the theorems demonstrated from them.

6. Conclusion

A theorem, for Aristotle, is a composite (σύνθετον) composed of elements. In *Metaphysics* Z17, Aristotle discusses the nature of composites (σύνθετα) that are unified not like a heap but like a syllable or like homoeomerous bodies such as flesh (1041b11–16). Presumably, theorems are composites of this latter sort, being unified not merely like

[108] Morrow 1970: 326–31.

[109] This is argued by Brown 1976: 259–69.

a heap. Aristotle argues that any such composite is something over and above the elements of which it is composed, something that is not itself an element (1041b16–33). He characterizes this entity as the substance of the composite and the first cause of its being (1041b27–8). Although he does not explicitly use the terminology of 'form', it is clear that the entity in question is the formal cause of the composite.[110] Thus, like syllables and flesh, a theorem has both material causes and a formal cause.[111] In the case of syllables, Aristotle identifies the formal cause with a mode of composition (σύνθεσις).[112] In the case of theorems, too, he seems to regard the formal cause as a kind of composition when he refers to the process of demonstrating a theorem as 'composing' (συντιθέναι, n. 97). Thus, the formal cause of a theorem is a mode of demonstration whereby the theorem is demonstrated from its ultimate material causes, the indemonstrable premises. Demonstration, on this account, is a kind of formal composition.

It should be noted that this kind of composition is distinct from the one whereby a proposition is composed of its syntactic constituents. In the first chapter of the

[110] Commentators agree that this entity is the form (εἶδος) of the composite (Ps.-Alex. Aphr. *in Metaph.* 542.12–543.26, Bonitz 1849: 360, Witt 1989: 116–17, Harte 2002: 133). As such, it is its formal cause (as introduced at *Phys.* 2.3 194b26–9 = *Metaph.* Δ 2 1013a26–9); see Moravcsik 1974: 7–9.

[111] In *Physics* 2.3, Aristotle states that for every example of material cause listed at 195a16–19, there is a corresponding formal cause (195a16–21 = *Metaph.* Δ 2 1013b17–23; see Alex. Aphr. *in Metaph.* 351.19–35, Zabarella 1601: ii.54–5, Wagner 1995: 464). Since hypotheses are said to be material causes of conclusions at 195a16–19, this implies that there is a corresponding formal cause of conclusions.

[112] *Metaph.* H 3 1043b4–14; cf. Alex. Aphr. *in Metaph.* 351.33, Bostock 1994: 262. Accordingly, σύνθεσις is listed as an example of formal cause at *Phys.* 2.3 195a21 (= *Metaph.* Δ 2 1013b22–3).

Analytics, Aristotle states that a premise is dissolved (διαλύεται) into a subject term and a predicate term.[113] Conversely, a premise is composed of these terms, and the terms are its parts.[114] Since parts are material causes of the whole of which they are parts, the subject and predicate terms are material causes of premises. Likewise, the major and minor terms of a demonstration are material causes of the conclusion.

In the *Analytics*, Aristotle defines a premise as an affirmation or denial in which the predicate term is affirmed or denied of the subject term.[115] He does not specify what kind of thing these affirmations and denials are, but a case can be made that they are a certain kind of linguistic expression, namely, sentences.[116] Sentences are composed of nouns and verbs, which are, in turn, composed of syllables and, ultimately, letters.[117] All these constituents are material causes of sentences. Consequently, they are also material

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[113] ὅρον δὲ καλῶ εἰς ὃν διαλύεται ἡ πρότασις, οἷον τό τε κατηγορούμενον καὶ τὸ καθ' οὗ κατηγορεῖται, *APr.* 1.1 24b16–17. Aristotle uses διαλύειν to refer to the resolution of a composite into its elements, e.g., of a syllable into letters, and of flesh into fire and earth (*Metaph.* Z 17 1041b11–16). As such, διάλυσις is the converse of σύνθεσις (*De caelo* 3.5 303b17–18, 3.6 304b25–33, 3.1 298b33–300a1, *Top.* 6.14 151a28; see Bonitz 1870: 184a11–16, b6–9).

[114] Alex. Aphr. *in APr.* 14.28–15.4; cf. Ammon. *in APr.* 22.18–22, Philop. *in APr.* 24.29–25.2.

[115] *APr.* 1.1 24a16–30, *APost.* 1.2 72a8–14.

[116] Crivelli and Charles 2011: 194, Crivelli 2012: 113–14, Malink 2015: 272. In *De interpretatione* 4 and 5, affirmations and denials are characterized as significant utterances. Similarly, they are characterized as linguistic expressions at *Cat.* 10 12b5–16.

[117] *Poet.* 20 1456b20–1457a28, *Int.* 2 16a19–26, 3 16b6–7, 4 16b26–32.

causes of the premises and conclusions of demonstrations, provided that these are sentences.

Clearly, however, they are material causes in a different sense from the one in which indemonstrable premises are material causes of the conclusions demonstrated from them.[118] One and the same demonstrable proposition can thus be viewed as the result of composition along two different dimensions: one syntactic and the other demonstrative.

The two modes of composition share some similarities. For instance, they both start from indivisible elements that are not themselves composite (the letters and indemonstrable premises, respectively). But there are also dissimilarities. For instance, in order to cognize a proposition one needs to cognize its syntactic constituents; if one does not cognize the terms 'triangle' and '2R', one does not cognize the proposition 'Every triangle has 2R'.[119] By contrast, in order to cognize a proposition one need not cognize its demonstrative constituents. Someone may cognize the proposition 'Every triangle has 2R' and know that it is true without cognizing any of the indemonstrable premises from which it is demonstrated. Although such a person does not have scientific knowledge of the proposition, she still cognizes it. In this respect, demonstrative composition does not

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[118] In *Posterior Analytics* 1.23, Aristotle describes the elements of conclusions as προτάσεις (84b22–8). He distinguishes them from the mental state through which they are grasped, namely, νοῦς (84b39–85a1). This suggests that the προτάσεις are not mental items, but rather linguistic expressions.

[119] 'How could one recognize speech if one did not know the syllables, or know these if one knew none of the letters (τῶν στοιχείων)?' *Protr.* B36.

resemble syntactic composition, but rather the kind of composition whereby homoeomerous bodies such as flesh are composed of elements such as fire and earth.[120] While fire and earth are constituents of flesh, they are not apparent to the eye. They can, however, be made apparent by separating them out in a suitable way:

> In flesh and wood and each thing of this sort, fire and earth are present potentially; for if these are separated out of those, they are apparent (φανερά). (*De caelo* 3.3 302a21–3; transl. Gill 1989: 78)

An analogous account can be given of the way in which indemonstrable premises are present in the propositions demonstrated from them. Thus, for example, Zabarella comments on Aristotle's discussion in *Posterior Analytics* 1.23:

> If the propositions of a demonstration are mediate, the first, immediate principles are not apparent in them (*in eis non apparent*), because they are in them only potentially. For a mediate proposition – as we said before with Aristotle – is not simple but composite, because it consists in some way of all the prior propositions through which it can be demonstrated, and it contains all of them potentially. (Zabarella 1608: 942; cf. 938)

[120] The last is one of the three kinds of composition (σύνθεσις) distinguished at *PA* 2.1 646a12–24; cf. *GA* 1.1 715a9–11, *Top.* 6.14 151a20–6.

In the case of homoeomerous bodies, fire and earth are brought from potentiality to actuality by a suitable physical process of separation. This process makes them 'apparent'. In the case of demonstrable propositions, the indemonstrable premises from which they are demonstrated are brought from potentiality to actuality by the demonstrator's cognitive act of identifying the right middle term(s). This act makes them 'apparent'. By thus grasping the middle terms and cognizing the indemonstrable premises to which they give rise, 'things which are potentially are discovered (εὑρίσκεται) when they are brought into actuality' (*Metaph.* Θ 9 1051a29–30).[121]

In *Physics* 1.4, Aristotle suggests that 'we know a composite (εἰδέναι τὸ σύνθετον) only when we know of which things and of how many things it consists' (187b11–13). With regard to the demonstrative composition of theorems, this is true provided that by 'know' is meant not mere cognition but scientific knowledge (ἐπιστήμη).

[121] The things that are said to be discovered in this passage are διαγράμματα (1051a22). These might be geometrical diagrams (Mendell 1984: 360 n. 3), constructions (Makin 2006: 13), demonstrations (Bonitz 1849: 407), or propositions (Heath 1949: 216; cf. Knorr 1975: 72). Whatever these things are taken to be, they are present potentially and are brought to actuality by an act of division that makes them apparent (φανερά, 1051a22–4). As such, they are analogous to the fire and earth present in homoeomerous bodies.

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